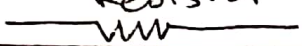
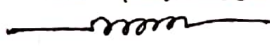
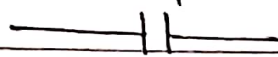


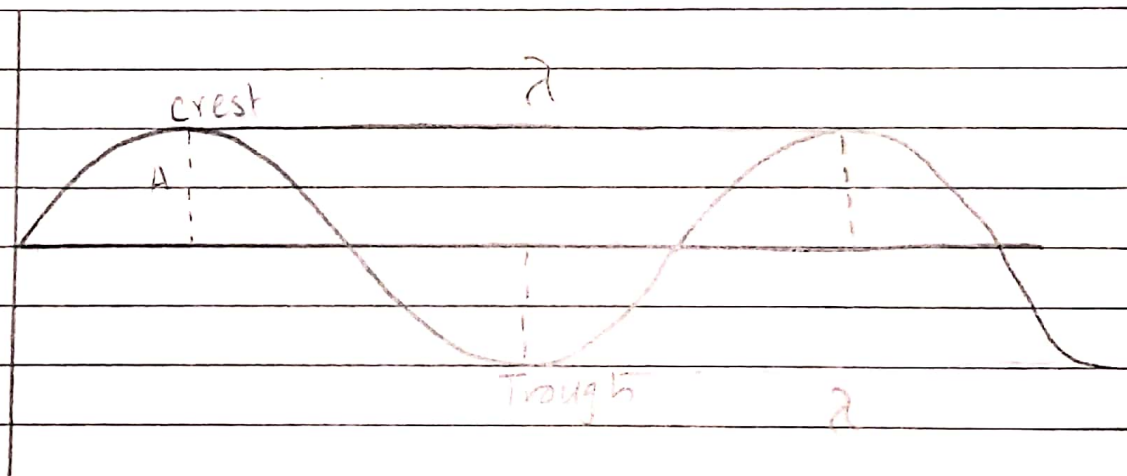
A C

- Rajiv Aryal

Single Phase AC Circuit Analysis

Device	Property	Unit
Resistor 	unit Resistance	Ohm
Inductor 	Inductance	Henny
Capacitor 	Capacitance	Faraday

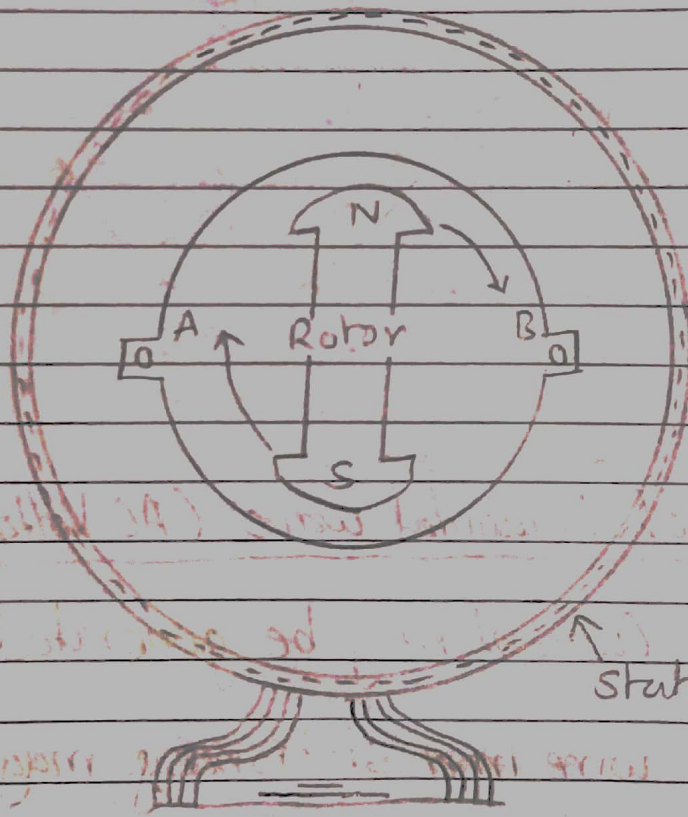
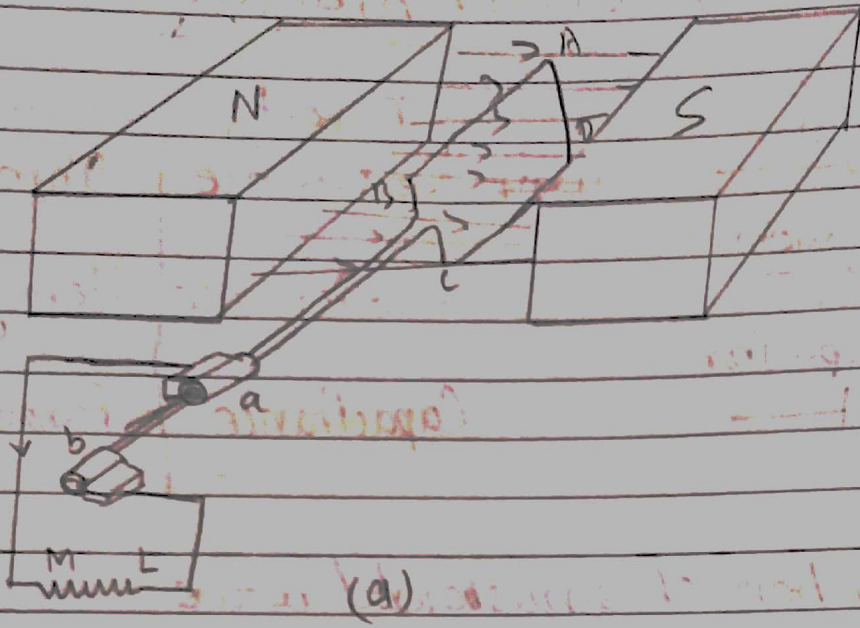
Generation of sinusoidal wave



Generation of Sinusoidal wave (AC Voltage Generation)

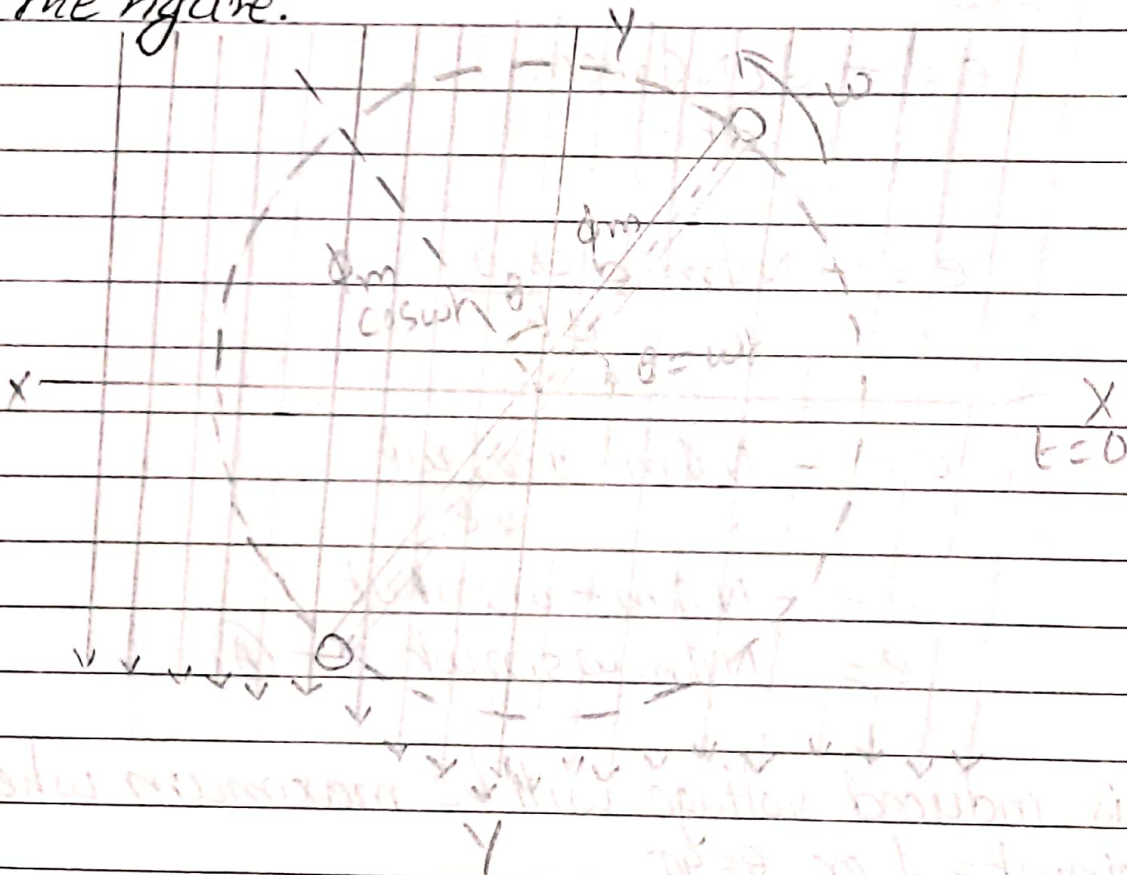
Alternative Current may be generated in two ways:-

- i) By rotating wire in a stationary magnetic field.
- ii) By rotating field within a stationary coil



Faraday's law states that whenever a conductor cuts the magnetic flux & EMF is generated. The value of voltage generated depends upon the no of turns of coil, strength of magnetic field & the speed at which coil ~~states~~ rotates (fig a) or magnet rotates (fig b).

Let us consider, a rectangular coil having 'n' turns and rotating in a uniform magnetic field with an angular velocity (ω) radian/sec as shown in the figure.



Here, ϕ_m is the magnetic flux linked with the coil. In time 't' sec the coil rotates through an angle $\theta = \omega t$. Add this deflected position the flux of the coil is $\phi_m \cos \omega t$ therefore for 'n' coils

$$\text{or, } N\phi$$

$$\text{or, } N\phi_m \cos \omega t$$

According to the ~~following~~ Faraday's law of electromagnetic induction in the coil is given by the rate of change of the flux linkage of the coil.

$$e = - \frac{d(N\phi)}{dt}$$

$$e = - \frac{d(N\phi_m \cos \theta)}{dt}$$

$$e = - N\phi_m \frac{d \cos \theta}{dt}$$

$$e = - N\phi_m \frac{d \cos \omega t}{dt}$$

$$e = - N\phi_m - \omega \sin \omega t$$

$$e = N\phi_m \omega \sin \omega t \quad \text{--- (A)}$$

This induced voltage will be maximum when $\sin \omega t = 1$ or $\theta = 90^\circ$

$$\text{i.e. } E_m = N\phi_m \omega$$

where; $E_m =$ maximum induced voltage

Substituting the value of $N\phi_m \omega$ in eqn A we get

$$e = E_m \sin \omega t$$

Now;

$$V = IR$$

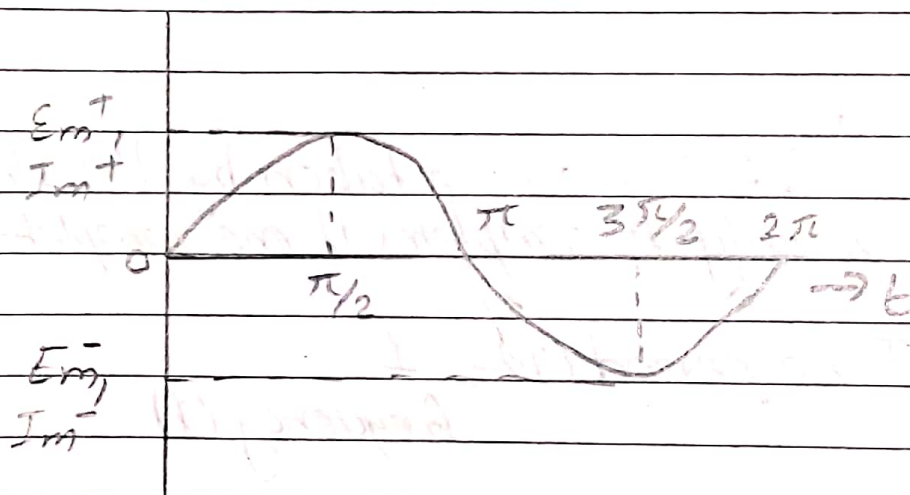
$$I = \frac{V}{R}$$

$$I = \frac{e}{R}$$

$$I = \frac{E_m \sin \omega t}{R}$$

$$I = I_m \sin \omega t \quad \left[\because \frac{E_m}{R} = I_m \right]$$

where; I_m is the maximum current.



Some sinusoidal Terminologies:-

1) Cycle:-

One complete set of positive and negative value of alternating quantity is known as cycle. Some times a cycle may be specified in terms of angular measure that is over 360 or 2π radian.

2) Amplitude or Peak values:-

It is the maximum positive or negative value of alternating quantity.

3) Time period:-

It is the time taken by the alternating quantity to complete its one complete cycle.

$$\text{Time period (T)} = \frac{1}{\text{frequency (f)}}$$

4) Frequency:-

frequency is the number of complete cycles that occur ~~in~~ in one second.

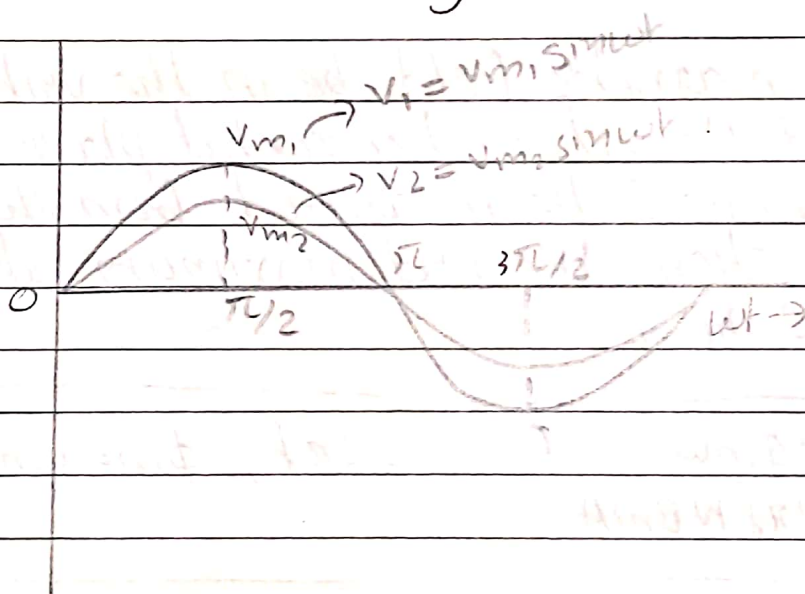
$$f = \frac{1}{T}$$

Its unit is cycle per second or Hertz.

Phase & Phase difference:-

Phase of an alternating quantity is defined as the fraction of time period of that alternating quantity which has elapsed or passed away since it last ~~period~~ passed from chosen position on origin.

Two alternating waves are said to be in phase when they reach their maximum values & zero values at the same time. Their peak value may be different in magnitude.



A square coil of 10cm side & 100 turns is rotated at a uniform speed of 1000 revolution / minutes. About an axis at right angles to a uniform magnetic field of 0.5 weber / m². Calculate the instantaneous value of induced emf. When the plane of coil is;

- i) At right angles to the field
- ii) In the plane of field

Soln

Let the magnetic field be in the vertical plane and the coil in horizontal plane. Also, let the angle θ be measured from the x-axis. Here we find maximum value of induced E.M.F. ;

$$E_m = N \phi_m \omega \quad \because \omega = 2\pi f, \quad \phi_m = B_m A$$

$$E_m = 2\pi f N B_m A$$

Now; frequency (f) = 1000 rev/min
 $= \frac{1000}{60} = 16.66 \text{ rev/sec}$

$$N = 100$$

$$B_m = 0.5 \text{ weber/m}^2$$

$$A = 0.1 \times 0.1 = 0.01 \text{ m}^2$$

i) In 1st case $\theta = 0^\circ$

$$E = E_m \sin \theta$$

$$E = 0$$

ii) In 2nd case $\theta = 90^\circ$

$$E = E_m \sin 90^\circ$$

$$= E_m$$

$$= 2\pi f N B_m A$$

$$= 2 \times 3.14 \times \frac{50}{3} \times 100 \times 0.5 \times 0.01$$

$$= 52.3 \text{ volt}$$

Average Value of current (I.AV)

The average value of an alternating current is expressed by that steady current (DC) which transfer across any circuit the same charge as is transferred by that alternating current during the same time. We know the standard equation of an AC current.

$$I = I_m \sin \theta$$

$$I_{avg} = \frac{1}{\pi} \int_0^\pi i d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} i m \sin \theta d\theta$$

$$= \frac{1}{\pi} i m \int_0^{\pi} \sin \theta d\theta$$

$$= \frac{i m}{\pi} \left[(-\cos \theta) \right]_0^{\pi}$$

$$= \frac{i m}{\pi} \left[(-\cos \pi) - (-\cos 0) \right]$$

$$= \frac{i m}{\pi} \left[\{-(-1)\} - (-1) \right]$$

$$= \frac{2 \cdot i m}{\pi}$$

$$I_{avg} = \frac{2 \cdot i m}{\pi}$$

$$\text{So, } I_{avg} = 0.637 i m //$$

Root Mean Square Value (r.m.s)

The r.m.s value is another factor for AC system. It is also known as effective or virtual value of A.C current. The r.m.s value of a AC is given by that steady current (DC) which when flowing through a given circuit for a given ~~time~~ times produces same heat as produced by an AC flowing through the circuit for the same time.

Derivation for r.m.s value:

We know, the standard form of sinusoidal AC is;
 $i = i_m \sin \omega t$

The mean of the squares of the instantaneous values of current over one complete cycle is;

$$= \int_0^{2\pi} \frac{i^2}{(2\pi - 0)} d\theta$$

The square root of this value is;

$$I_{rms} = \sqrt{\int_0^{2\pi} \frac{i^2}{2\pi} d\theta}$$

$$= \sqrt{\int_0^{2\pi} \frac{(i_m \sin \theta)^2}{2\pi} d\theta}$$

$$= \int_0^{2\pi} \frac{im^2}{2\pi} \sin^2 \theta \, d\theta$$

$$= \int_0^{2\pi} \frac{im^2}{2\pi} \frac{1 - \cos 2\theta}{2} \, d\theta$$

$$= \int_0^{2\pi} \frac{im^2}{4\pi} (1 - \cos 2\theta) \, d\theta$$

$$= \frac{im^2}{4\pi} \left[2\pi - 0 \right] - \left[\frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

$$= \frac{im^2}{4\pi} \left[2\pi - \left(\frac{\sin 2 \cdot 2\pi}{2} \right) - \left(\frac{\sin 2 \cdot 0}{2} \right) \right]$$

$$= \frac{im^2}{4\pi} (2\pi - 0)$$

$$= \frac{im^2}{2\pi}$$

$$= \frac{im}{\sqrt{2}}$$

$$i_{rms} = 0.707 i_m$$

Hence, we find that, rms value of current is equal $0.707 \times$ max value of current.

Form factor:-

It is define as the ratio;

$$K_f = \frac{\text{rms value}}{\text{average value}}$$

$$= \frac{0.707 \times i_m}{0.637 \times i_m}$$

$$K_f = 1.11$$

Amplitude factor:-

It is define as the ratio

$$K_a = \frac{\text{maximum value}}{\text{r.m.s value}}$$

$$K_a = \frac{i_m}{\frac{i_m}{\sqrt{2}}}$$

$$K_a = 1.414$$

(Q) For the maximum given below. Find maximum value, Effective value. Average value, form factor & Amplitude factor?

$$E = 12 \sin 28\pi t$$

Comparing this with $E = E_m \sin \omega t$ and we get

$$E_m = 12$$

$$\omega = 28\pi$$

$$r_{ms} = 0.707 \times 12 = 8.484 \text{ V}$$

$$E_{avg} = 0.637 \times 12 = 7.644 \text{ V}$$

$$K_f = 1.11$$

$$K_a = 1.414$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

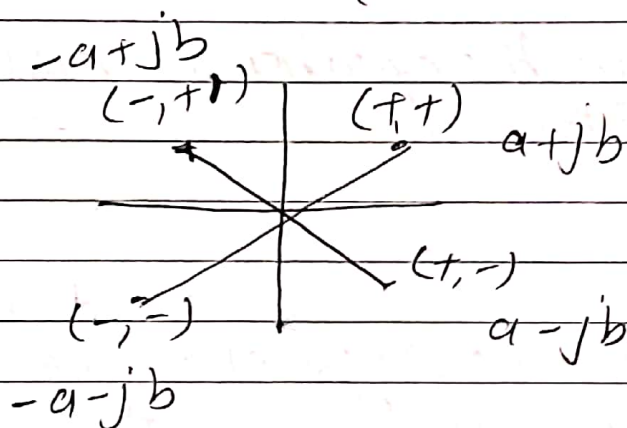
$$= \frac{28\pi}{2\pi}$$

$$= 14 \text{ Hz}$$

Complex number

A complex number contains a real part and imaginary part in the form of $E = a + jb$. A magnitude is $|E| = \sqrt{a^2 + b^2}$ & angle with x-axis is given by;

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$



Q) Perform the following operation:

$$\begin{aligned} z &= (8 + 6j) \times (-10 - 7.5j) \\ &= 8(-10 - 7.5j) + 6j(-10 - 7.5j) \\ &= -80 - 60j - 60j - 45j^2 \\ &= -80 + 45 - 120j \\ &= -35 - 120j \end{aligned}$$

$$a = -35$$

$$b = -120$$

$$= \sqrt{(-35)^2 + (-120)^2}$$

$$= \sqrt{15625}$$

$$= 125$$

$$\phi = \tan^{-1}\left(\frac{-120}{-35}\right) = 73.73 + 180 = 253.73^\circ$$

① Trigonometry form of vector:

The complex vector $E = a + jb$ can be written in the form of $E = E_m (\cos \theta \pm j \sin \theta)$

$$E = a + jb$$

$$E = E_m (\cos \theta \pm j \sin \theta)$$

② Exponential form

In Exponential form the equation becomes

$$E = E_m e^{\pm j\theta}$$

③ Polar form

The equation $E = E_m (\cos \theta \pm j \sin \theta)$ can be written in polar form as $E = E_m \angle \pm \theta$

Write the Equivalent exponential & polar form of vectors:

$$3 + 4j$$

$$E_m = \sqrt{3^2 + 4^2}$$

$$= 5$$

$$\theta = \tan^{-1} \left(\frac{4}{3} \right)$$

$$= 53.13$$

In Exponential form

$$E = 5e^{53.13j}$$

In Polar form

$$E = 5 \angle 53.13$$

In trigonometry form

$$E = 5 (\cos 53.13 + j \sin 53.13)$$

③ Perform the following operation & Express the final result in polar form.

$$5 \angle 30 + 8 \angle -30$$

$$5 (\cos 30 + j \sin 30) + 8 (\cos (-30) + j \sin (-30))$$

$$5 \times \frac{\sqrt{3}}{2} + 5 \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{2} + 8 \times \frac{(-1)}{2}$$

$$\frac{13\sqrt{3} - 3j}{2}$$

$$\frac{13\sqrt{3}}{2} - \frac{3j}{2}$$

$$11.25 - 1.5j$$

$$E_m = \sqrt{(11.25)^2 + (1.5)^2}$$

$$= 11.34$$

$$\theta = \tan^{-1} \left(\frac{-1.5}{11.25} \right)$$

$$= -7.6$$

So, $E = 11.35 \angle -7.6$

Let $A = a_1 + jb_1 = A e^{j\theta_1}$
 $B = a_2 + jb_2 = B e^{j\theta_2}$

$A \cdot B = A B \cdot e^{j\theta_1} \cdot e^{j\theta_2} = A B e^{(\theta_1 + \theta_2) j}$

$A \cdot B = A B \angle (\theta_1 + \theta_2)$

$\frac{A}{B} = \frac{A \angle \theta_1}{B \angle \theta_2} = \frac{A}{B} \angle (\theta_1 - \theta_2)$

Perform the following operation in polar form

$(8+6j) \cdot (-10-7.5j)$

$\sqrt{64+36} \quad \sqrt{100+56.25}$

$A = 10 \quad B = 12.5$

$\theta = \tan^{-1} \left(\frac{6}{8} \right)$

$\theta = \tan^{-1} \left(\frac{-7.5}{-10} \right)$

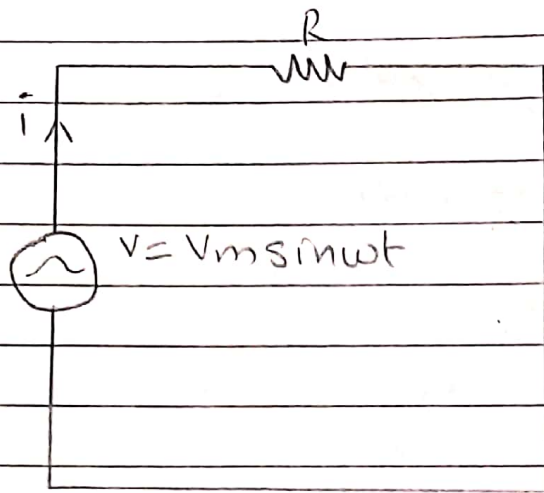
$\theta_1 = 36.86$

$\theta_2 = 36.86$

$A \cdot B = A \cdot B \angle (\theta_1 + \theta_2)$
 $= 10 \times 12.5 (36.86 + 36.86)$
 $= 125 \angle 73.76^\circ$

① AC through pure resistance circuit

Let us consider the circuit shown as below:-



Here, the applied voltage is given by the equation;

$$V = V_m \sin \theta = V_m \sin \omega t \quad \text{--- ①}$$

Let, R be the ohmic resistance,
 i be the instantaneous current

According to the ohm's law the voltage drop is equal to the applied voltage.

$$V = iR$$

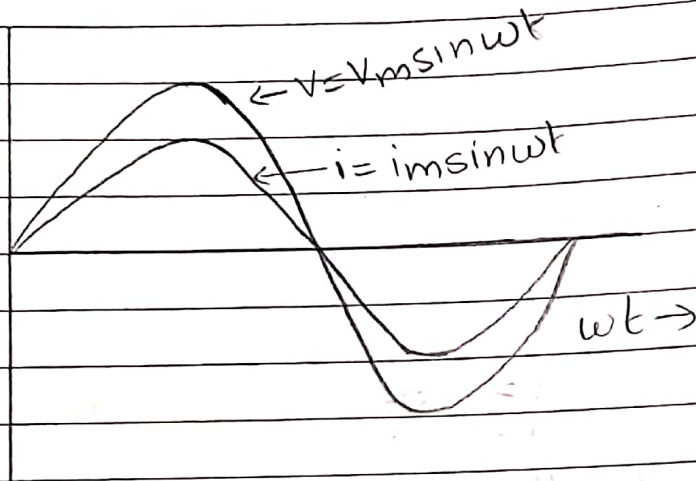
Putting the value of V from eqn ①

$$\text{or, } V_m \sin \omega t = iR$$

$$\text{or, } i = \frac{V_m \sin \omega t}{R}$$

$$\text{or, } i = i_m \sin \omega t \quad \text{--- ②}$$

From eqn ① & eqn ② we find that the alternating voltage and current are in phase with each other.



Now;

$$\text{Power} = VI$$

$$= V_m \sin wt \times i_m \sin wt$$

$$= V_m \cdot i_m \sin^2 wt$$

$$= V_m \cdot i_m \left(\frac{1 - \cos 2wt}{2} \right)$$

$$= \frac{V_m \cdot i_m}{2} - \frac{V_m i_m \cos 2wt}{2}$$

So, the power consists of two parts.

$$\frac{V_m i_m}{2} - \frac{V_m i_m \cos 2wt}{2}$$

↑

constant
part

↑

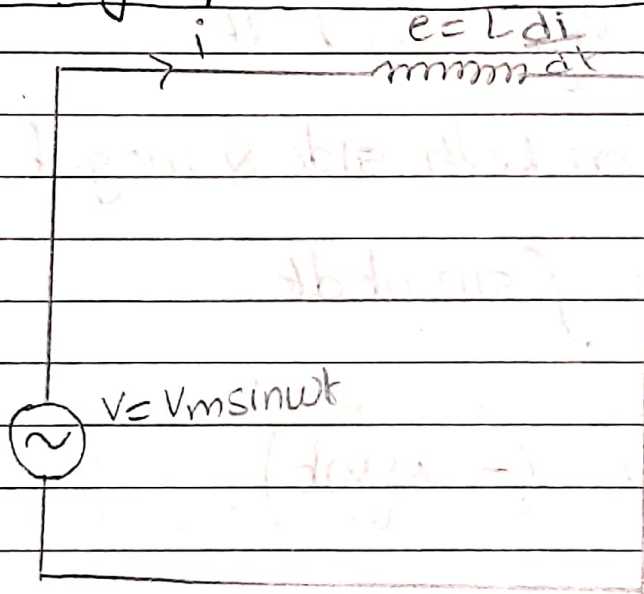
fluctuating
part

For a complete cycle the average value of the fluctuating power is always zero. Hence, the power is

$$P = \frac{V_m i_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}}$$

$$P = V_{rms} \cdot I_{rms}$$

⑪ AC through pure inductive circuit



$$V = -e$$

Whenever an alternating voltage is applied to a purely inductive coil a back emf is produced. This back emf at every instant opposes the rise or fall of current through the coil. This back emf is given by;

$$e = -L \frac{di}{dt}$$

Since, this induced emf is equal to the source voltage

$$\text{or, } V = -e$$

$$\text{or, } V_m \sin \omega t = - \left(-L \frac{di}{dt} \right) = L \frac{di}{dt}$$

$$\text{or, } V_m \sin \omega t = L \frac{di}{dt}$$

$$\text{or, } di = \frac{V_m}{L} \sin \omega t dt$$

Integrating on both side & we get

$$\int di = \frac{V_m}{L} \int \sin \omega t dt$$

$$i = \frac{V_m}{L} \left(-\frac{\cos \omega t}{\omega} \right)$$

$$i = \frac{V_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

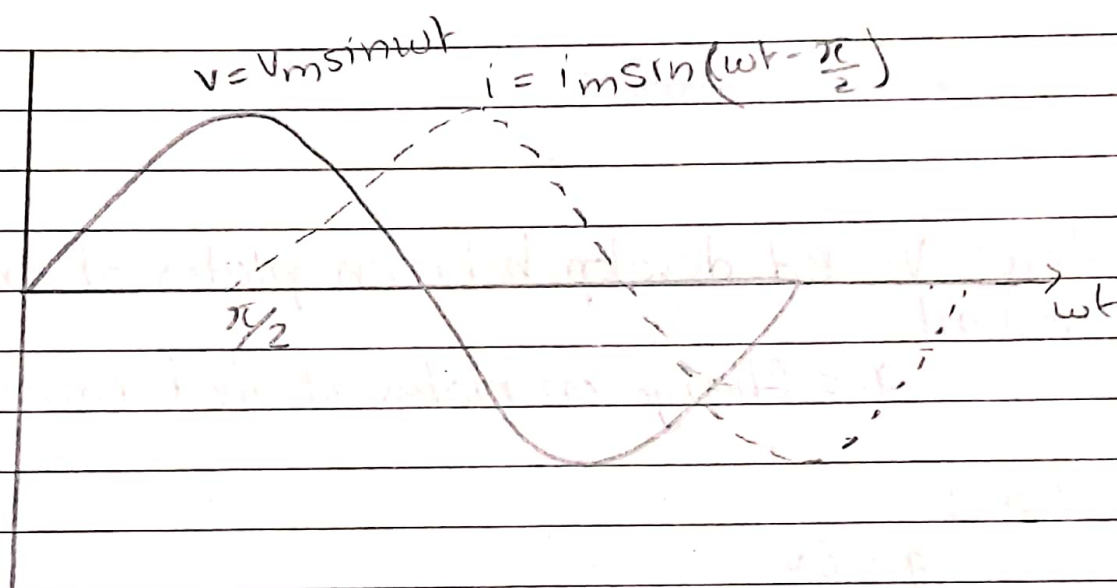
$$i = \frac{V_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$\omega L = X_L$$

$$i = \frac{V_m}{X_L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

where; $i_m = \frac{V_m}{X_L}$

$$i = i_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

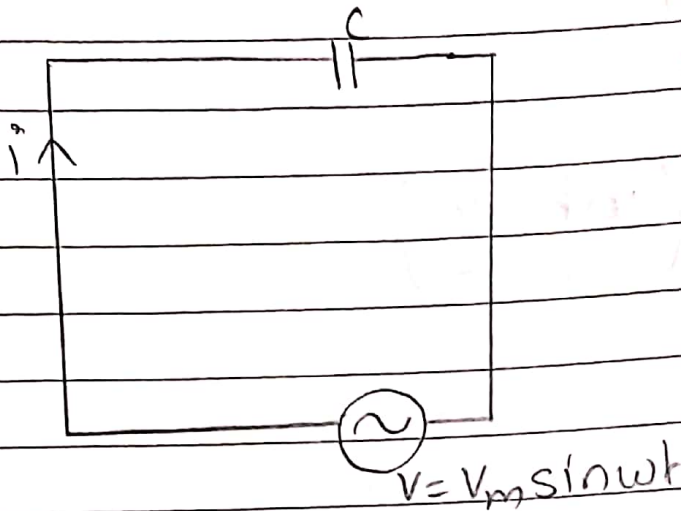


Here; X_L is the resistance known as inductive reactance.

$$X_L = \omega L = 2\pi fL$$

AC through pure Capacitance

When an alternating voltage is applied to the plate of the capacitor. The capacitor is charged first in one direction and then in opposite direction.



Here, V = P.d develop between plates at any instant.

q = Charge on plates at that instant.

Now;

$$q = Cv$$

C = Capacitance of capacitor
(faraday)

$$q = C V_m \sin \omega t \quad [\because V = V_m \sin \omega t]$$

Now, Current ' i ' is given by the rate of flow of charge

$$i = \frac{dq}{dt}$$

$$i = \frac{d(C V_m \sin \omega t)}{dt}$$

$$i = C V_m \frac{d \sin \omega t}{dt}$$

$$i = C V_m \omega \cos \omega t$$

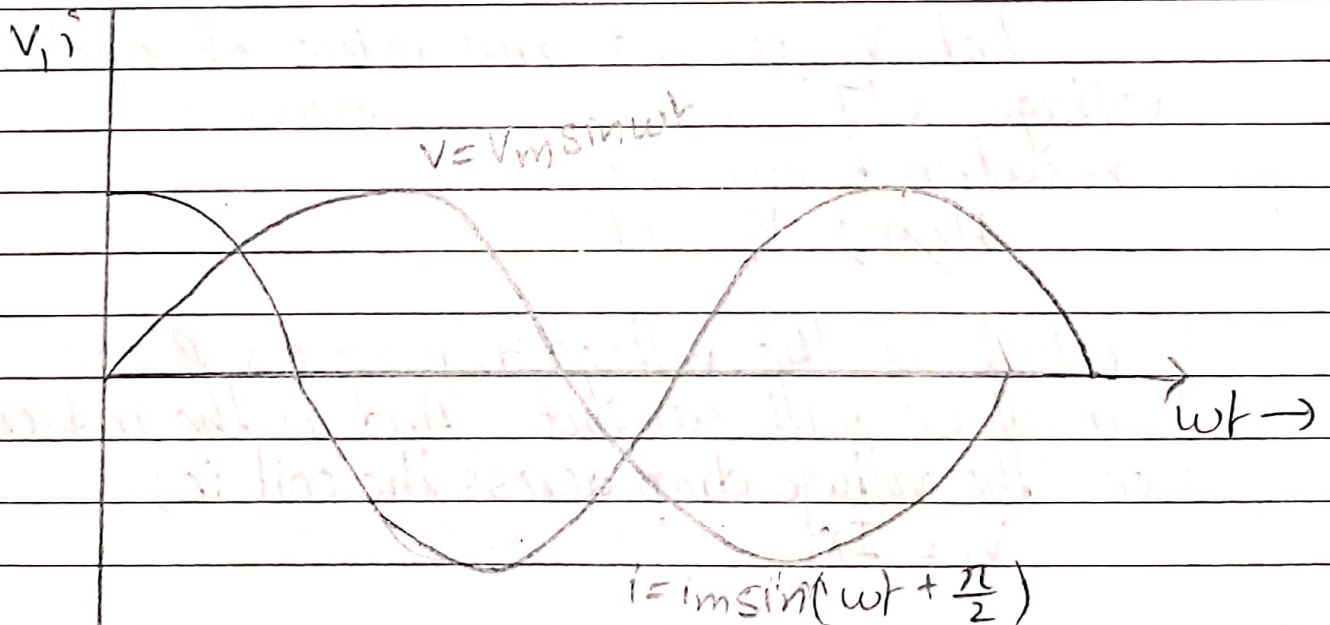
$$i = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

$$i = \frac{V_m \cos \omega t}{X_C} \left\{ \left[\because \frac{1}{\omega C} = X_C \right] \right. \left. \left[X_C = \frac{1}{\omega C} \text{ is known as capacitive reactance} \right] \right.$$

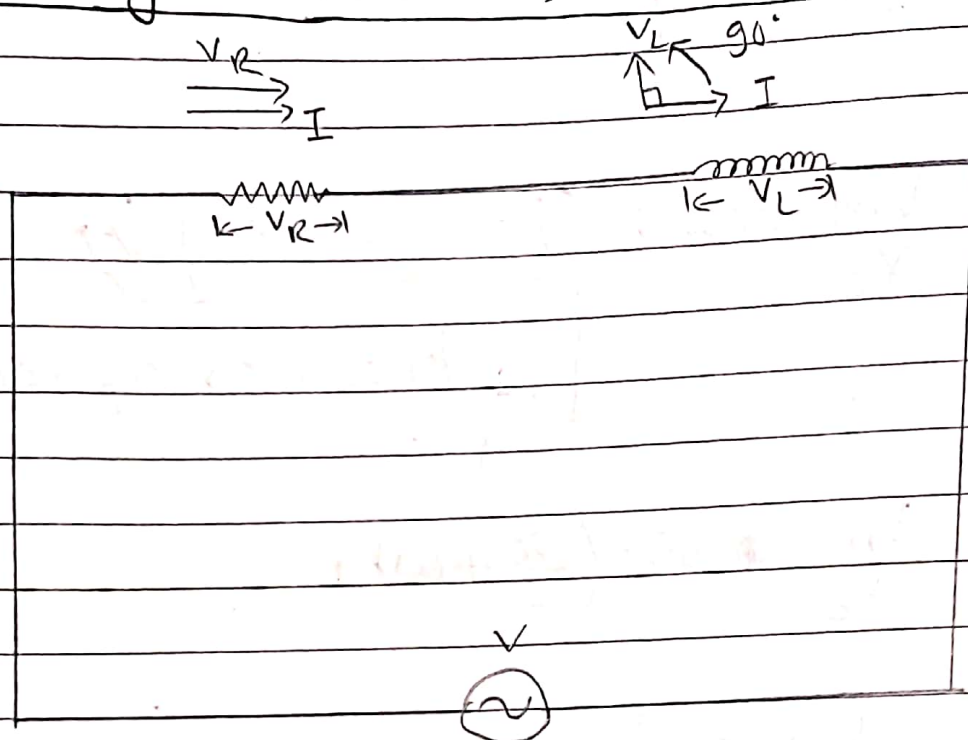
$$i = \frac{V_m}{X_C} \sin \left(\frac{\pi}{2} + \omega t \right)$$

Here; $\frac{V_m}{X_C} = i_m$

$$i = i_m \sin \left(\frac{\pi}{2} + \omega t \right)$$



AC Through resistance & inductance



A pure resistance R and a pure inductive coil of inductance ' L ' are shown in connected in series in the figure above.

Let ' V ' is equal rms value of applied voltage & ' I ' is equal rms value of resultant current:

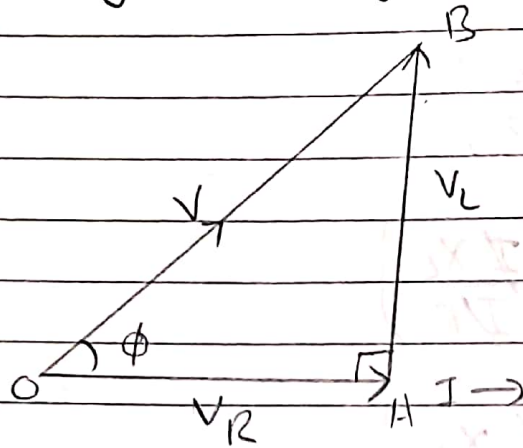
$$\text{Here; } V_R = IR$$

which is the voltage drop across R . i.e. in phase with current. And in the inductance the voltage drop across the coil is;

$$V_L = IX_L$$

This voltage drop is ahead of current by 90° .

Now, showing the voltages in vector form:



In $\triangle OAB$, $\vec{OA}$ represents ohmic voltage drop V_R , $\vec{AB}$ represent inductive voltage drop V_L . The applied voltage V is the vector sum of the two.

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\therefore V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= \sqrt{I^2 (R^2 + X_L^2)}$$

$$V = I \sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

Here, the quantity $\sqrt{R^2 + X_L^2}$ is known as impedance of the circuit (Z).

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z^2 = R^2 + X_L^2$$

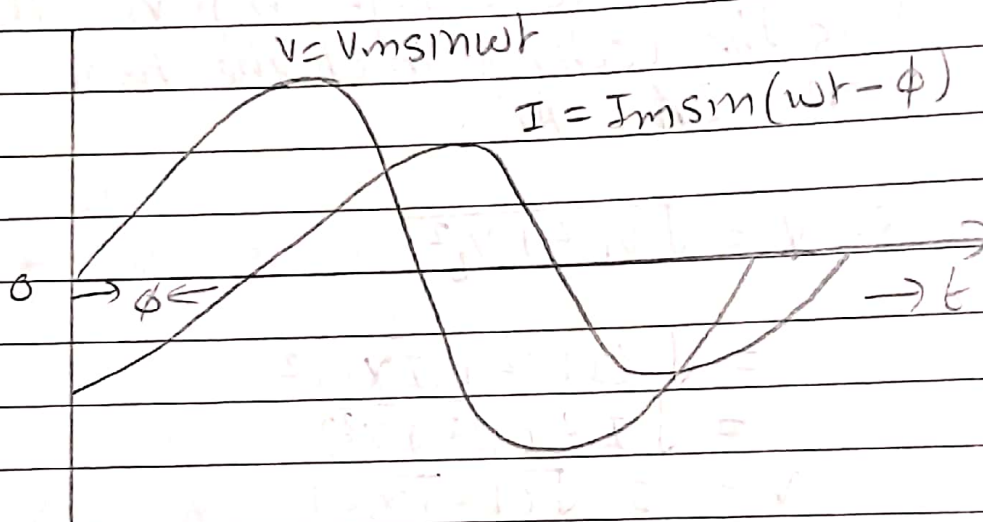
From the figure a voltage triangle, it is clear that the applied voltage V leads the current I by an

angle ' ϕ '.

$$\phi = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

$$\phi = \tan^{-1} \left(\frac{I \cdot X_L}{I/R} \right)$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$



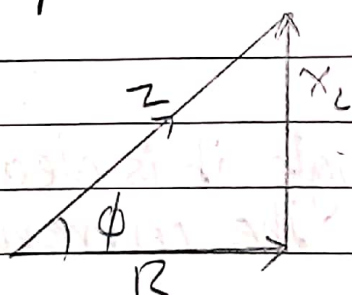
Average Power

$$P = V_{rms} \cdot I_{rms} \cos \phi$$

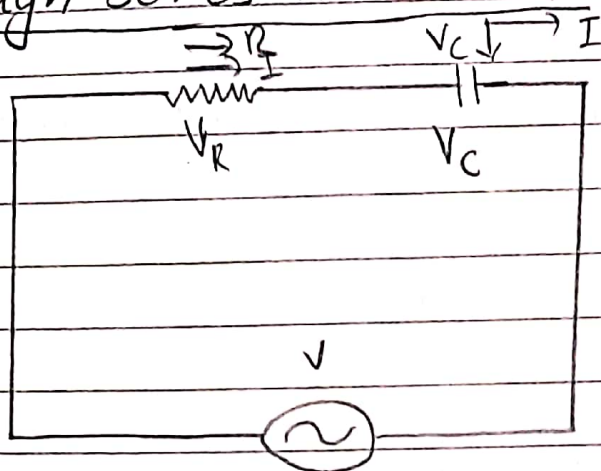
Alternative,

$$Z = \sqrt{R^2 + X_L^2}$$

Impedance triangle,



AC Through series R.C Circuit



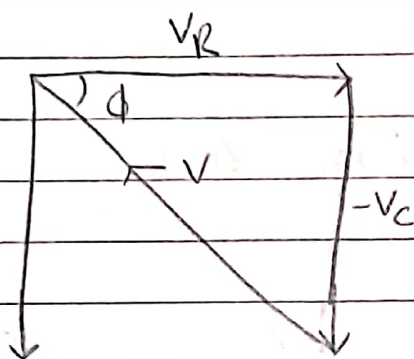
Here, in the figure V_R is the voltage drop across the resistance 'R' which is in phase to the current I . In the figure V_C is the

voltage drop across capacitor lagging current ' I ' by 90° .

$$V_R = IR \quad \text{--- (1)}$$

$$V_C = I X_C \quad \text{--- (2)}$$

Fig (11)



So, the voltage triangle

is given by above in fig (11)

Now;

$$\text{or } V = \sqrt{V_R^2 + (-V_C)^2}$$

$$\text{or } V = \sqrt{V_R^2 + V_C^2}$$

$$\text{or } V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\text{or } V = I \sqrt{R^2 + X_C^2}$$

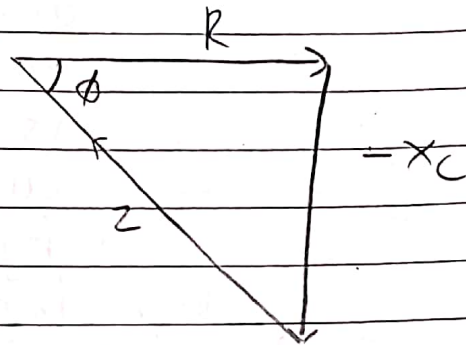
$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$\text{Let; } Z = \sqrt{R^2 + X_C^2}$$

$$\text{i.e. } I = \frac{V}{Z}$$

where, z is known as the impedance of the circuit.

Impedance triangle for the circuit:



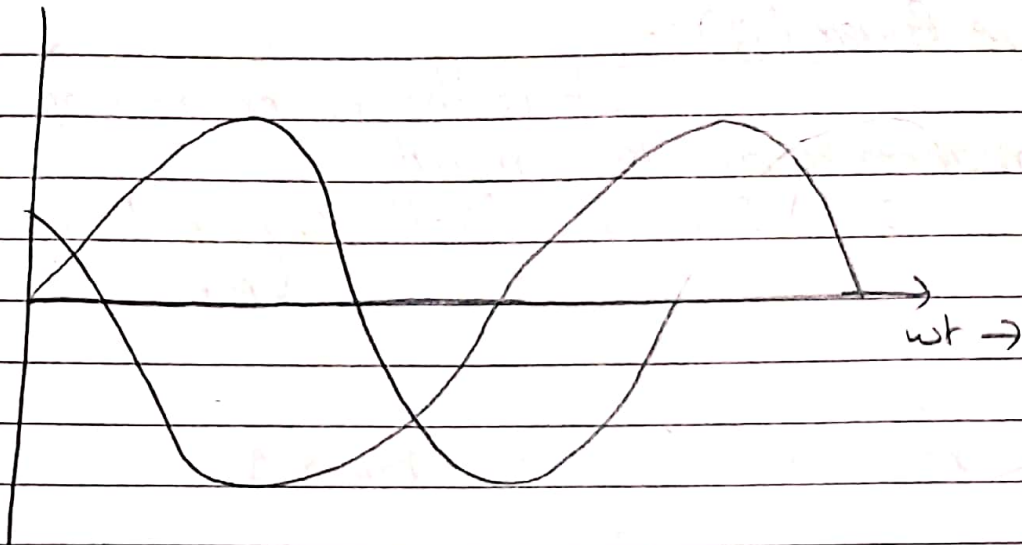
From the impedance Δ is;

$$\tan \phi = \frac{-X_c}{R}$$

$$\phi = \tan^{-1} \left(\frac{-X_c}{R} \right)$$

From the voltage triangle, it is found that the current leads the voltage by an angle ϕ such that

$$\tan \phi = \frac{-X_c}{R}$$



Power

① Apparent Power (s)

It is given by the product of rms values of applied voltage and circuit current

$$\begin{aligned} \therefore S &= VI \\ &= IZ \cdot I \\ &= I^2 Z \end{aligned}$$

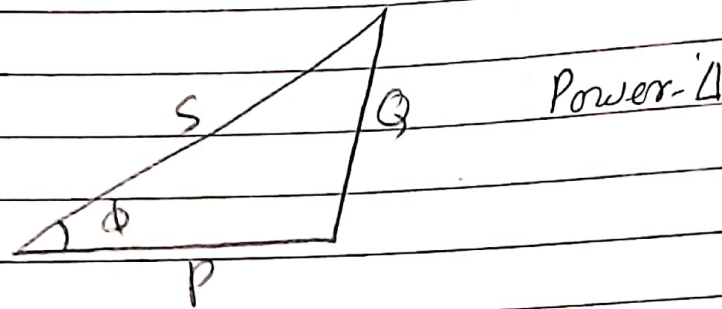
Its ~~value~~ unit is volt-ampere.

② Active Power (W or P)

It is a power which is actually dissipated in the circuit resistance.

$$P = I^2 R = VI \cos \phi \text{ (watts)}$$

- ③ Reactive Power (Q):- It is the power developed in the reactants of the circuit.
 $Q = VI \sin \phi$ (Volt-Ampere-Reactive)
 VAR



$$S^2 = P^2 + Q^2$$

- ④ Power factor (P.f.):
 It is defined as;

- (i) ~~As~~ the cosine of the angle of lag or lead. ($\cos \phi$)
- ② The ratio of resistance to impedance $= \frac{R}{Z}$
- ③ The ratio of true power to apparent power =

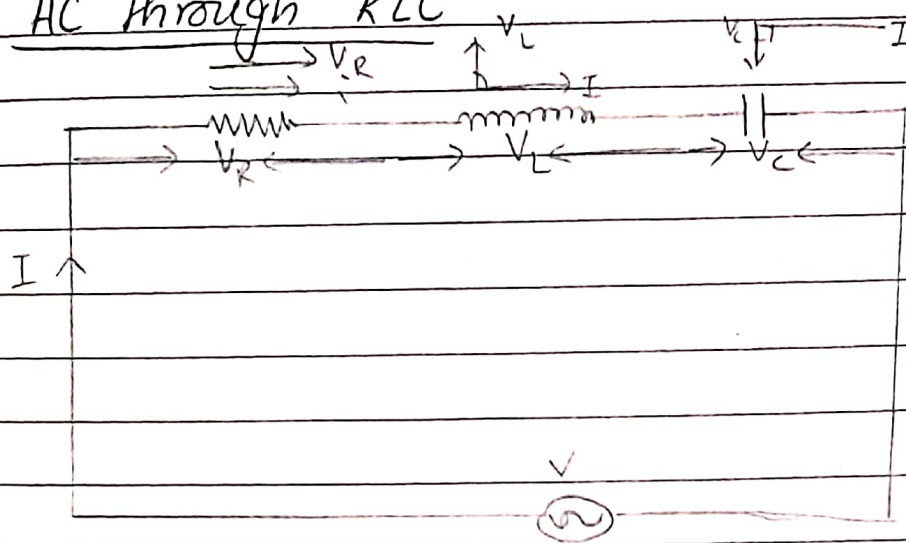
$$\frac{\text{True power}}{\text{Apparent power}}$$

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$$P.F = \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + X_C^2}} \quad \Bigg| \quad = \frac{R}{\sqrt{R^2 + X_L^2}}$$

AC Through RLC



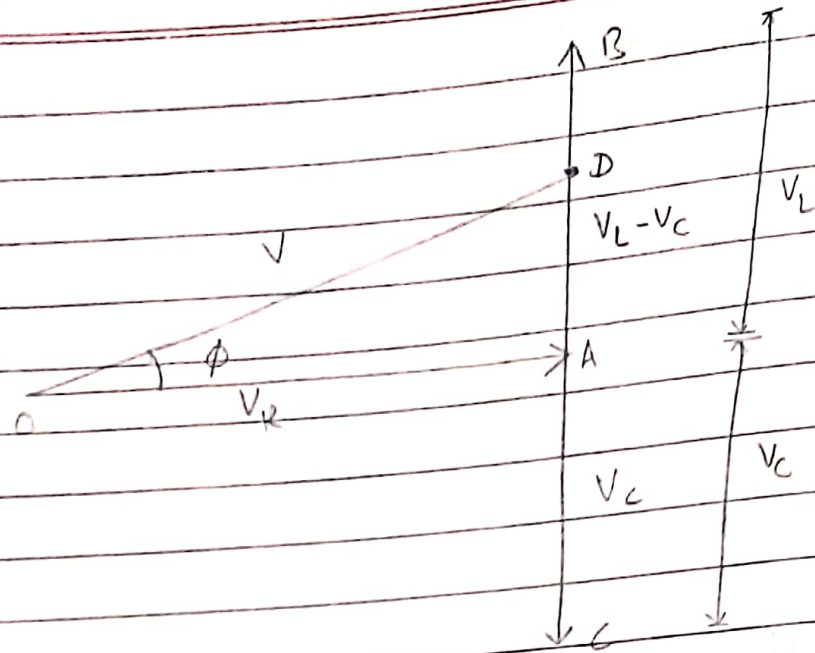
The following figure is the combination of resistor, inductor & capacitor in series connection. with an AC supply of r.m.s voltage 'V'.

Let, $V_R = IR$ (Voltage drop across resistance R .
 V_R is in phase with I)

$V_L = I X_L =$ (Voltage drop across inductance L .
 V_L is leading I by $\frac{\pi}{2}$)

$V_C = I X_C =$ (Voltage drop across capacitance C .
 V_C is lagging I by $\frac{\pi}{2}$)

Now, We draw the voltage triangle ;



OA represents V_R . AB represents V_L . AC represent V_C .

Now,

Here, V_L & V_C are in out phase with each other. So the resultant is AD. The resultant triangle is OAD.

$$OD = \sqrt{OA^2 + AD^2}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + \{(IX_L) - (IX_C)\}^2}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

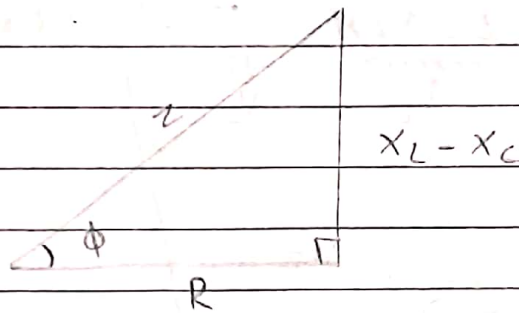
$$m \quad I = \frac{V}{Z}$$

where; Z is the total impedance of the circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

Now, the phase angle is given by



$$\tan \phi = \frac{X_L - X_C}{R}$$

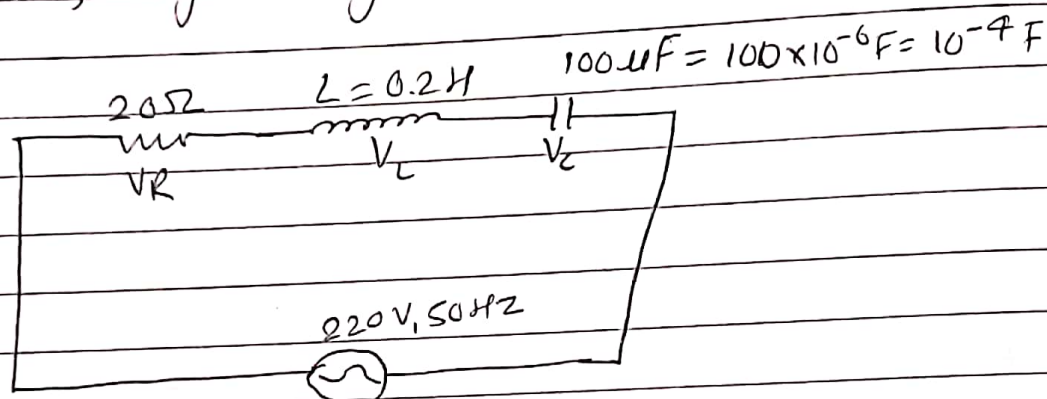
Now;

Power factor:

$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

A resistance of 20Ω , ~~ind~~ inductance of $0.2H$ & capacitance of $100\mu F$ in series across ~~200~~ $220V$ & $50Hz$ mains. Determine the following

- 1) Impedance
- 2) Current
- 3) Voltage across R, L, C
- 4) Power in watts & VA
- 5) P.F & Angle of lag.



$$\textcircled{1} \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 37\Omega$$

$$R = 20\Omega$$

$$\omega = 2\pi f$$

$$= 2 \times 3.14 \times 50 = 314$$

$$X_L = \omega L = 314 \times 0.2 = 63$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 10^{-4}} = 32$$

$$\textcircled{11} \quad V = IZ$$

$$I = \frac{V}{Z} = \frac{220}{37} = 6A$$

③ $V_R = IR = 120V$

$V_L = I \times L = 6 \times 63 = 378V$

$V_C = I \times C = 192V$

④ Power in watts

$$\begin{aligned} P &= VI \cos \phi \\ &= 220 \times 6 \times 0.54 \\ &= 712.8 \\ &= 713 \text{ W} \end{aligned}$$

⑤ Power Factor

$\cos \phi \text{ P.f.} = 0.54$

Power in VA

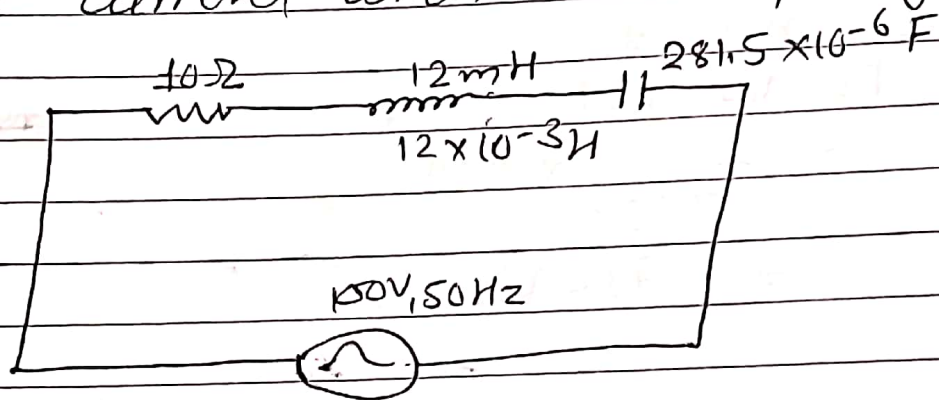
$= 220 \times 6 = 1320$

Angle of lag

$$\begin{aligned} \cos \phi &= 0.54 \\ \phi &= \cos^{-1}(0.54) \\ \phi &= 57.31^\circ \end{aligned}$$

RLC

A coil of resistance 10 ohm and inductance 12 mH and Capacitance $281.5 \text{ } \mu\text{F}$ in series. The supply voltage is 100 V . calculate the value of the current when the frequency is 50 Hz .



$$I = \frac{V}{Z}$$

R

$$\omega = 2\pi f$$

$$X_L = \omega L$$

$$= 2 \times 3.14 \times 50$$

$$= 3768 \times 10^{-3} = 3.768$$

$$= 3.768$$

$$= \sqrt{100}$$

$$Z = \sqrt{(10)^2 + (3.768 - 11.31)^2} \times C = \frac{1}{\omega C}$$

$$= \frac{100}{12.52}$$

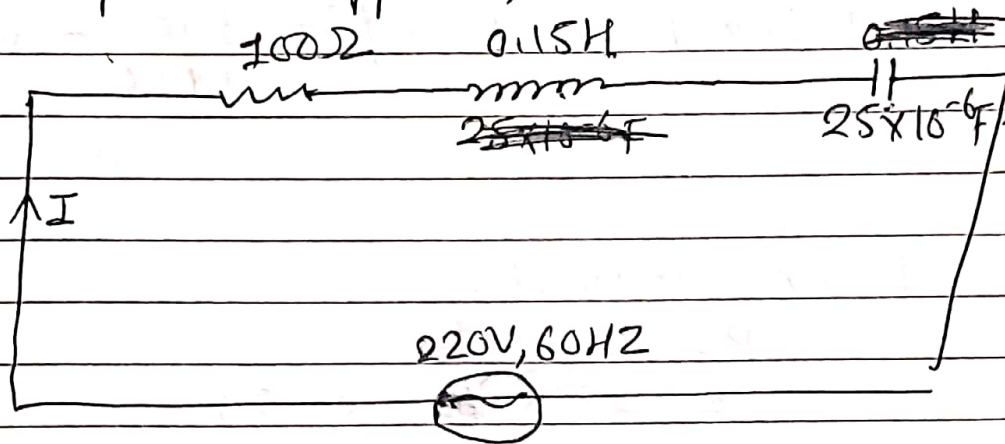
$$= \frac{1}{314 \times 2.815 \times 10^{-6}}$$

$$= 7.98 \text{ A} \approx 8 \text{ A}$$

$$= 11.31$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- ④ A series circuit with a resistor 100Ω , Capacitor $25 \mu F$ and Inductance $0.15 H$ is connected across $220 V$, $60 Hz$ supply. Calculate current, the power supplied, Power factor.



$$R = 100 \Omega$$

$$C = 25 \times 10^{-6} F$$

$$L = 0.15 H$$

$$V = 220 V$$

$$F = 60 Hz$$

$$\omega = 2\pi f = 2 \times 3.14 \times 60 = 376.8 = 377 \text{ rad/sec}$$

Calculating X_L

Calculating X_C

$$\begin{aligned} X_L &= \omega L \\ &= 377 \times 0.15 \\ &= 56.55 \Omega \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{\omega C} \\ &= \frac{1}{377 \times 25 \times 10^{-6}} \end{aligned}$$

$$= 106.1 \Omega$$

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(100)^2 + (56.55 - 106.1)^2} \\ &= 111.6 \, \Omega \end{aligned}$$

$$I = \frac{V}{Z} = \frac{220}{111.6} = 1.97 \, A$$

$$\text{Power factor } (\cos \phi) = \frac{100}{111.6} = 0.89$$

Power supplied

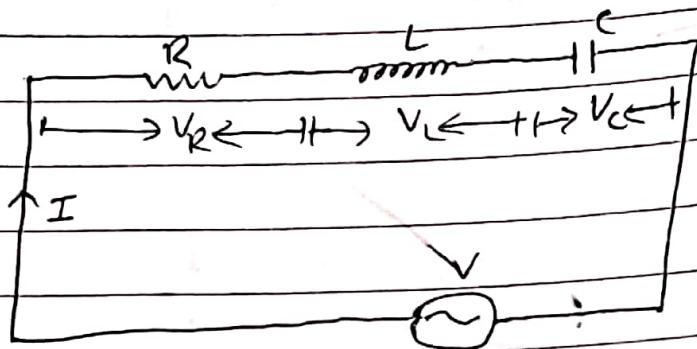
$$\begin{aligned} P &= VI \cos \phi \\ &= 220 \times 1.97 \times 0.89 \\ &= 385.726 \, \text{watt} \end{aligned}$$

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Resonance in RLC series circuit



From the figure, the net reactants and the ^{total} impedance is;

$$X = X_L - X_C$$

$$Z = \sqrt{R^2 + X^2}$$

In this circuit, An AC source of constant voltage V is applied with variable frequency from zero to infinity.

There would be a certain of the applied voltage which would make

$X_L = X_C$ in magnitude. In that case $X = 0$ & $Z = R$. This is called electrical resonance and this frequency is said to be resonance frequency.

Calculation of resonance frequency;

For a circuit in electrical resonance we know that the net reactants is zero.

$$X = 0$$

$$X_L - X_C = 0$$

$$or, X_L = X_C$$

$$or, \omega L = \frac{1}{\omega C}$$

$$or, \omega^2 LC = 1$$

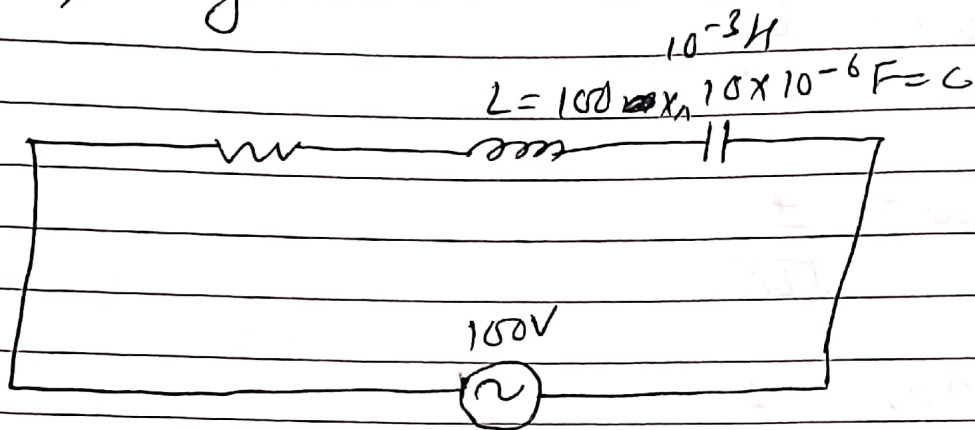
$$or, \omega^2 = \frac{1}{LC}$$

$$or, \omega = \frac{1}{\sqrt{LC}}$$

$$or, 2\pi f = \frac{1}{\sqrt{LC}}$$

$$or, \boxed{F = \frac{1}{2\pi\sqrt{LC}}} \quad \#$$

- ④ An RLC series circuit consist of a resistance of 1000Ω , an inductance of ~~milliH~~ 10^{-3} H and capacitance of $10 \mu\text{F}$. If a voltage of 100 V is applied across the circuit. Find the resonance frequency.



Calculating the resonance frequency

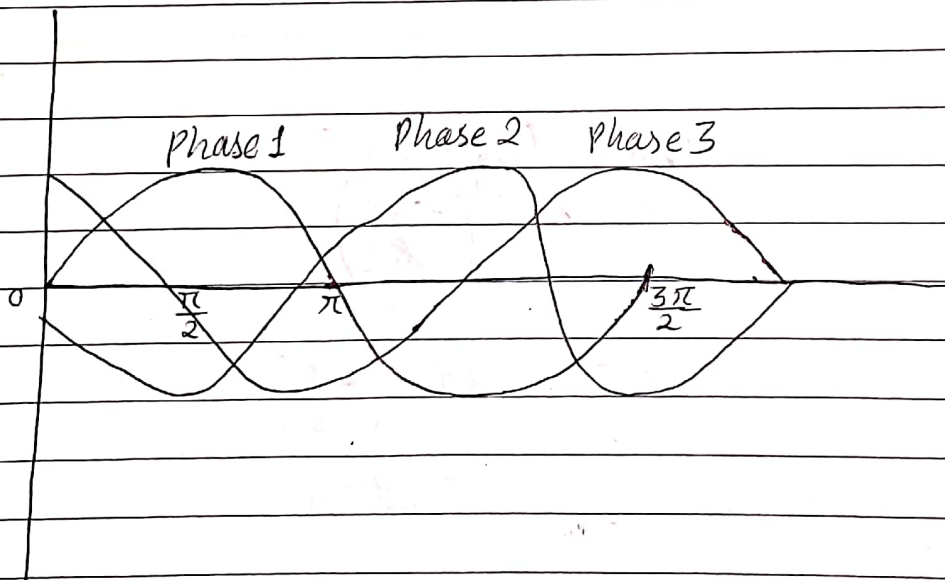
$$f = \frac{1}{2 \times 3.14 \sqrt{1000 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$$f = 159.23 \text{ Hz}$$

Three Phase System :-

The system which has three phases i.e. the current will pass through three wires. And there will be one neutral wire for passing the fault current to the earth is known as 3 phase system. #

The sum of the line currents in the three phase system is equal to zero & their phases are differentiated at an angle of 120° . The three phase system has 4 wires, i.e. the current carrying conductor and the one neutral. The current in the neutral wire is equal to the sum of the line current of three wires.



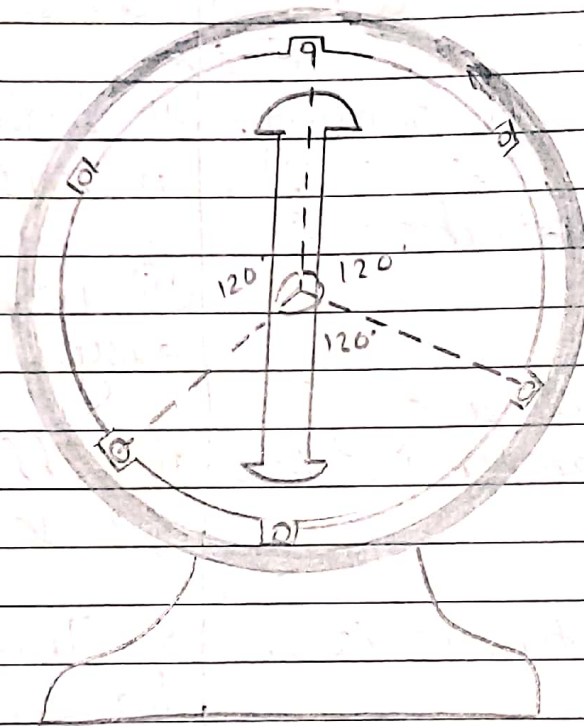
The 120° phase difference of the three phases is must for the proper working of the system. Otherwise, the system becomes damage.

Advantages of 3 phase system (Polyphase) over single phase system

Single phase	Three phase
① Power delivered is pulsating phase	① Power delivered is constant
② Single induction motors are not self starting.	② 3 phase induction motors are self-starting
③ Parallel operation is not easy.	③ Parallel operation is easy.
④ Efficiency of a single phase motor is less.	④ Efficiency of a 3 phase motor is high.
⑤ Single phase motor have pulsating torque (τ).	⑤ Three phase motor have uniform torque (τ).
⑥ Single phase motors have lower power factor.	⑥ Three phase motors have higher power factor.

* Generation of 3-phase Sinusoidal Voltage:-

Let V_R , V_Y & V_B are the voltages generated by a three phase alternator which has three armature coils rr' , yy' & bb' displaced 120° apart from each other. The figure represents the alternator to present 3 three phase voltage.



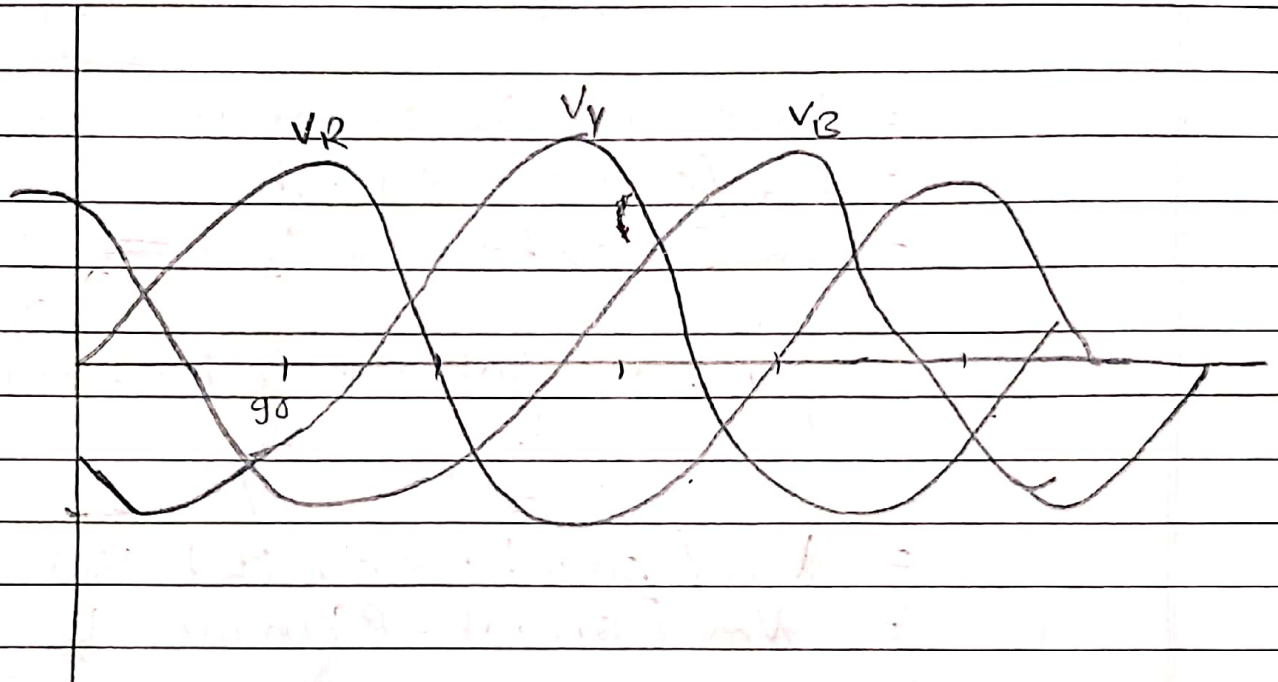
Assuming the ~~wave~~ waves to be sinusoidal we get the following wave equations; we get;

$$V_R = V_m \sin \omega t \quad \text{--- (I)}$$

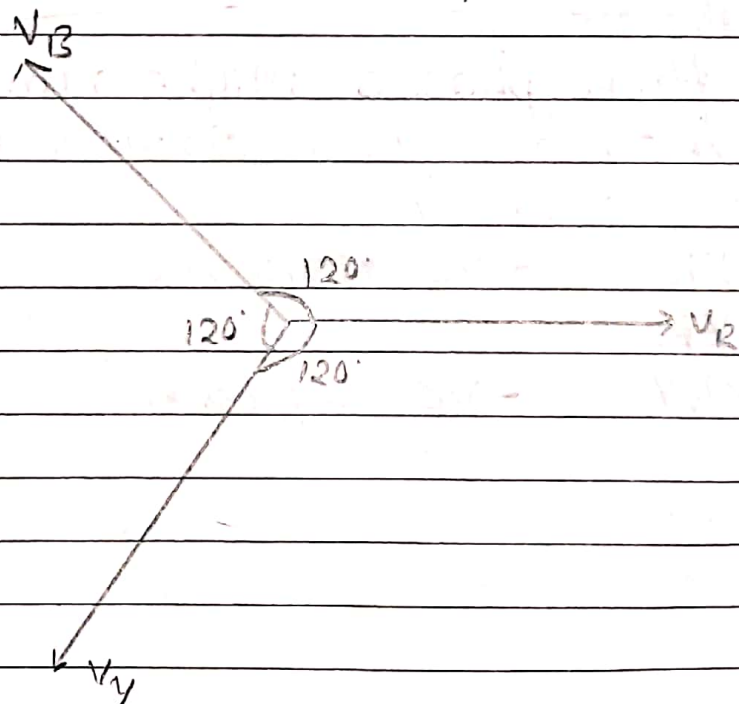
$$V_Y = V_m \sin(\omega t - 120^\circ) \quad \text{--- (II)}$$

$$V_B = V_m \sin(\omega t - 240^\circ) \quad \text{--- (III)}$$

The system is symmetrical where the voltages are of same magnitude and same frequency and are displaced 120° from each other.



Phasor diagram:-



Here; the sum of the equations ①, ② and ③ is zero. Therefore, the resultant instantaneous e.m.f. is given by

$$= V_r + V_y + V_b$$

$$= V_m \sin \omega t + V_m \sin(\omega t - 120) + V_m \sin(\omega t - 240)$$

$$= V_m [\sin \omega t + \sin(\omega t - 120) + \sin(\omega t - 240)]$$

$$= V_m \left[\sin \omega t + 2 \sin \left(\frac{\omega t - 120 + \omega t - 240}{2} \right) \cdot \cos \left(\frac{\omega t - 120 - \omega t + 240}{2} \right) \right]$$

$$= V_m [\sin \omega t + 2 \sin(\omega t - 180) \cdot \cos 60]$$

$$= V_m [\sin \omega t - 2 \sin \omega t \cdot \frac{1}{2}]$$

$$= V_m \times 0$$

$$= 0$$

Note: The three phases may be numbered 1, 2, 3. And the colours used commercially are red, yellow and blue.

R Y B +ve sequence

~~R Y B~~

R B Y -ve sequence

Interconnection of three phases:

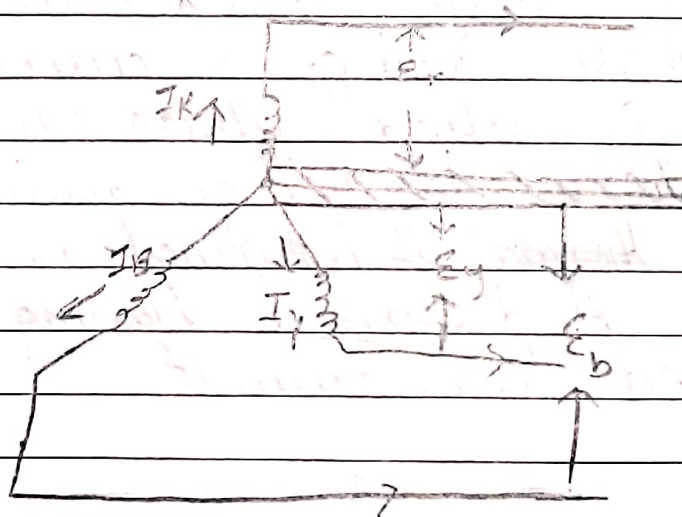
The three phases are generally interconnected which results in substantial saving of copper wire. The general method of interconnection are:

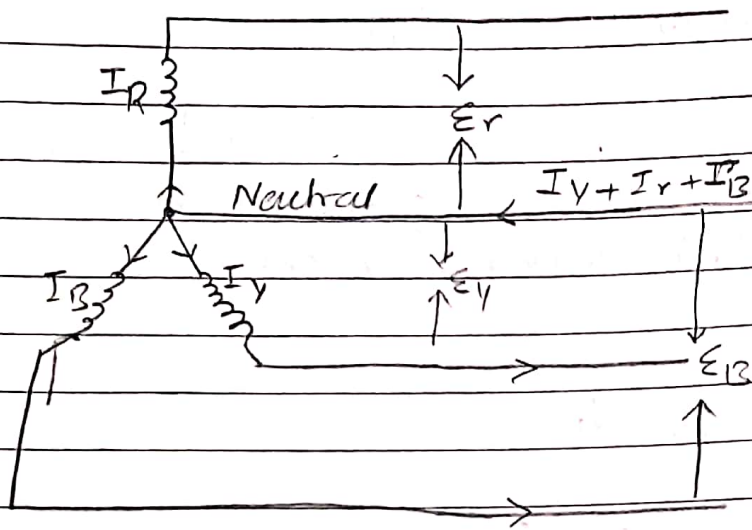
- (a) Star or (Y) connection
- (b) Mesh or Delta (Δ) Connection

(a) Star or (Y) connection :

In this method of interconnection the similar ends of three coils are joint together at point 'N'.

The figure below represents the star connection.

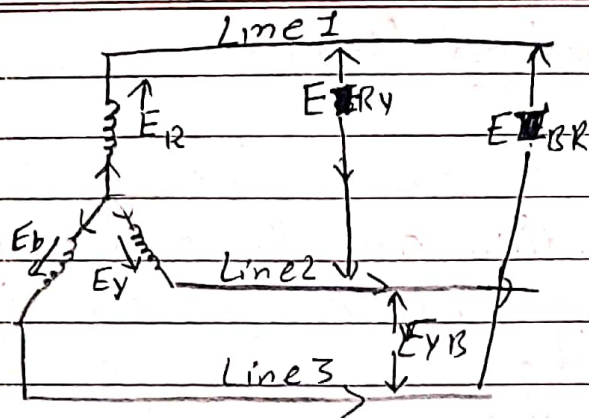




The point 'N' is known as star point or neutral point. If this three phase voltage system is applied across a balance symmetrical load, the neutral wire will be carrying three currents which are exactly equal in ~~mag~~ magnitude but are 120° out of phase with each other. therefore; the vector is $I_Y + I_B + I_r = 0$ which is zero.

The current I_r , I_b and I_y is known as phase current. The voltage induced in each winding is called phase voltage & current in each winding is called phase current.

However, the voltage available between any pairs of ~~terminal~~ terminals is called line voltage. And current flowing in each line is called line current.



$$\begin{aligned} E_{RY} &= E_R - E_Y \\ E_{BR} &= E_B - E_R \\ E_{YB} &= E_Y - E_B \end{aligned}$$

Line current = phase current

$$I_L = I_{ph}$$

$$\cancel{V_L = V_{ph}}$$

$$\begin{aligned} \text{line voltage} &= \sqrt{3} \text{ phase voltage} \\ V_L &= \sqrt{3} V_{ph} \end{aligned}$$

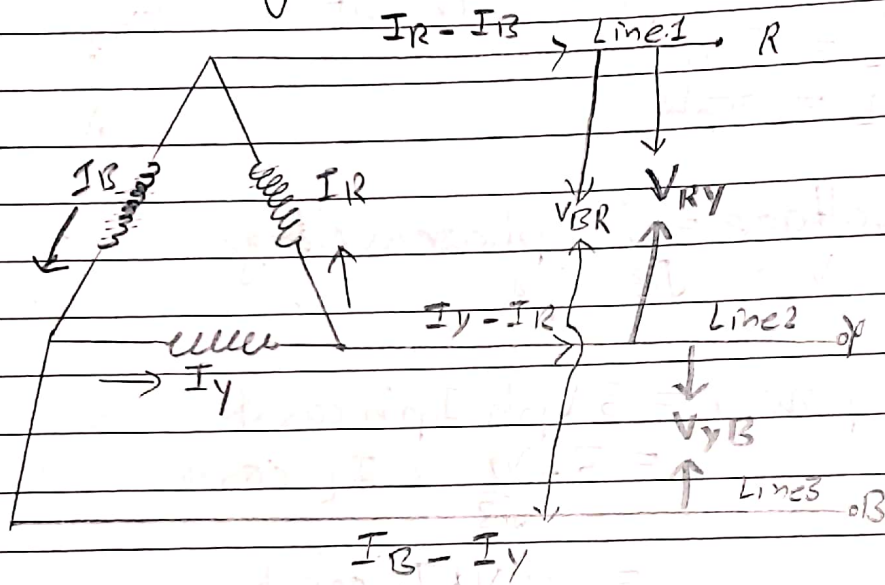
$$\begin{aligned} (P) \text{ Active power} &= 3 V_{ph} I_{ph} \cos \phi \\ &= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos \phi \\ &= \sqrt{3} V_L I_L \cos \phi \end{aligned}$$

$$(Q) \text{ Reactive power} = \sqrt{3} V_L I_L \sin \phi$$

$$(S) \text{ Apparent power} = \sqrt{3} V_L I_L$$

Delta Connection (Mesh)

In this form, of interconnection the dissimilar ends of 3 phase winding are joint together ie. the starting ending of one phase is joint to the finishing end of other phase. The following in the figure of delta connection.



If the system is balance then the sum of three voltages round the close mesh is zero, Hence, no current ie V_B, V_Y, V_R can flow around mesh when the terminals are opened.

In the figure, there is ~~no~~ ~~ph~~ only one phase winding completely include in any point of terminals. Therefore, the voltage between any pairs of lines is equal to the phase voltage of the phase winding connected between two lines.

Here, V_{RY} lead V_{YB} by 120° , V_{YB} leads V_{BR} by 120° .

Let $V_{RY} = V_{BR} = V_{YB} = V_L$ and $V_L = V_{ph}$.

Current in lines:

$$I_1 = I_R - I_B$$

$$I_2 = I_Y - I_R$$

$$I_3 = I_B - I_Y$$

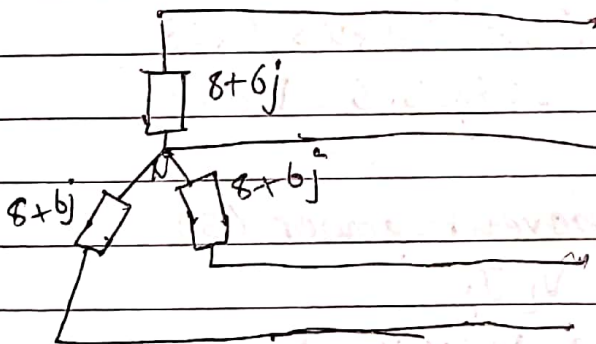
Also;

$$I_L = \sqrt{3} \cdot I_{ph}$$

$$\boxed{\text{Line current} = \sqrt{3} \text{ phase current}}$$

$$\text{Power (P)} = \sqrt{3} V_L I_L \cos \phi$$

- ③ A balanced star connection load of $8 + 6j \Omega$ per phase is connected to a balanced 3 phase 400 V supply. Find the line current, power factor, power and total voltage ampere?



$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

$$Z_{ph} = \sqrt{8^2 + 6^2}$$

$$Z_{ph} = 10 \Omega$$

Resistance (R) = Real part

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$\frac{400}{\sqrt{3}} = V_{ph}$$

$$V_{ph} = 231$$

$$V = \sqrt{3} Z$$

$$V_{ph} = I_{ph} Z_{ph}$$

$$231 = I_{ph} 10$$

$$I_{ph} = 23.1 \text{ A} = I_L$$

$$\text{Power factor } (\cos \phi) = \frac{R}{Z}$$

$$= \frac{R_{ph}}{Z_{ph}}$$

$$= \frac{8}{10}$$

$$= 0.8$$

$$\text{Active power (P)} = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi$$

$$= \sqrt{3} \times 400 \times 23.1 \times 0.8$$

$$= 12803.3 \text{ Watt}$$

$$\text{Voltage Ampere} = \text{Apparent power (S)}$$

$$= \sqrt{3} V_L I_L$$

$$= \sqrt{3} \times 400 \times 23.1$$

$$= 16004.1 \text{ VA}$$

$$S = \sqrt{P^2 + Q^2}$$

$$S^2 = P^2 + Q^2$$

$$Q^2 = S^2 - P^2$$

$$Q^2 = \sqrt{(16504.1)^2 - (12803.3)^2}$$

$$Q = 9602.43$$

Q An AC current given by $i = 14.14 \sin(\omega t + \pi/6)$

Find the rms value of current and phase angle in degree

$$\text{phase angle} = \frac{\pi}{6}$$

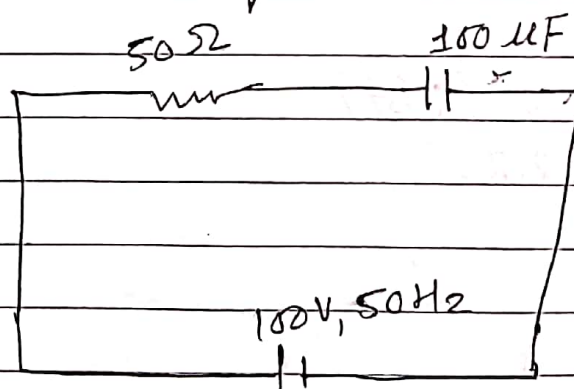
$$i_{\text{rms}} = \frac{i_m}{\sqrt{2}}$$

$$= \frac{14.14}{\sqrt{2}}$$

$$i_{\text{rms}} = \frac{14.14}{\sqrt{2}} = 9.99 \approx 10$$

$$= 30^\circ$$

Q A pure resistance of 50Ω is in series with the pure capacitance of $100 \mu\text{F}$. Series combination is connected across 100 V , 50 Hz . Find impedance, current, power factor, phase angle, voltage across resistor (V_R), voltage across capacitor (V_C)?



$$R = 50 \Omega$$

$$C = 100 \times 10^{-6} \text{ F}$$

$$V = 100 \text{ V}$$

$$F = 50 \text{ Hz}$$

$$\omega = 2\pi f$$

$$= 2 \times 3.14 \times 50$$

$$= 314$$

$$X_L = \frac{1}{\omega C}$$

$$= \frac{1}{314 \times 100 \times 10^{-6}}$$

$$= 31.84$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{50^2 + (31.84)^2}$$

$$= \sqrt{2500 + 1013.78}$$

$$= 59.27$$

$$V = IZ$$

$$100 = I \times 59.27$$

$$I = \frac{100}{59.27}$$

$$I = 1.68 \text{ A}$$

$$\cos \phi = \frac{R}{Z}$$

$$= \frac{50}{59.27}$$

$$= 0.84$$

$$\phi = \cos^{-1}(0.84)$$

$$= 32.85^\circ$$

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$$V_R = IR$$

$$= 1.68 \times 50$$

$$= 84 \text{ V}$$

$$V_C = IX_C$$

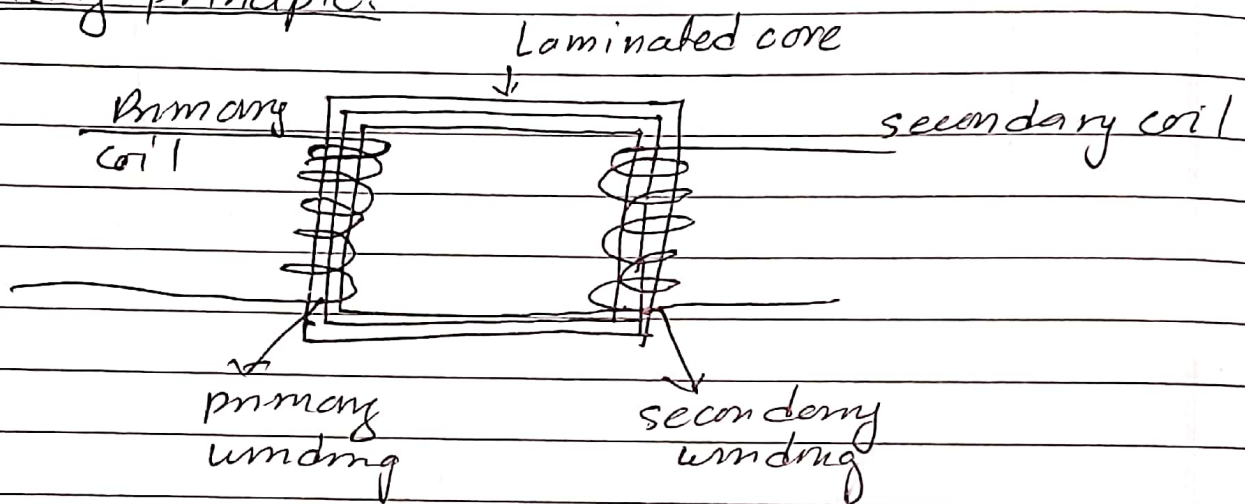
$$= 1.68 \times 31.84$$

$$= 53.49 \text{ V}$$

Transformer :-

A transformer is a device that transfers electrical energy from one circuit to another by electromagnetic induction. The electrical energy is always transferred without the change in frequency but may involve changes in the magnitude of voltage and current.

Working principle:



The main principle of operation of a transformer is mutual inductance between the two circuit which is linked by a common magnetic flux (Φ).

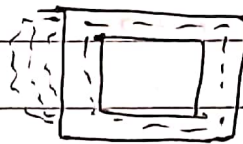
A basic transformer consists of two coils that are electrically separate and inductive but are magnetically linked. ~~through~~ The two coils have high mutual inductance. If one coil is connected to a source of ~~the~~ single phase AC voltage, an alternating flux is set up in the laminated core which produces mutually induced emf.

In the secondary winding. The first coil is called primary winding and second ~~is~~ coil is called secondary winding.

Construction:

The transformer has following parts:-

(a) The core:-



The core provides the path for the magnetic lines of flux. There are 2 ~~sets~~ shape:

~~two~~ laminated steel core and Hollow core

(b) Winding:-

The transformer consists of two coils called windings. The winding connected to source is called primary winding and the winding that is connected to the load is called secondary winding.

~~Let~~ Let

N_A = no of turns in primary

N_B = no of turns in secondary

E_A = voltage in primary coil

E_B = voltage in secondary coil

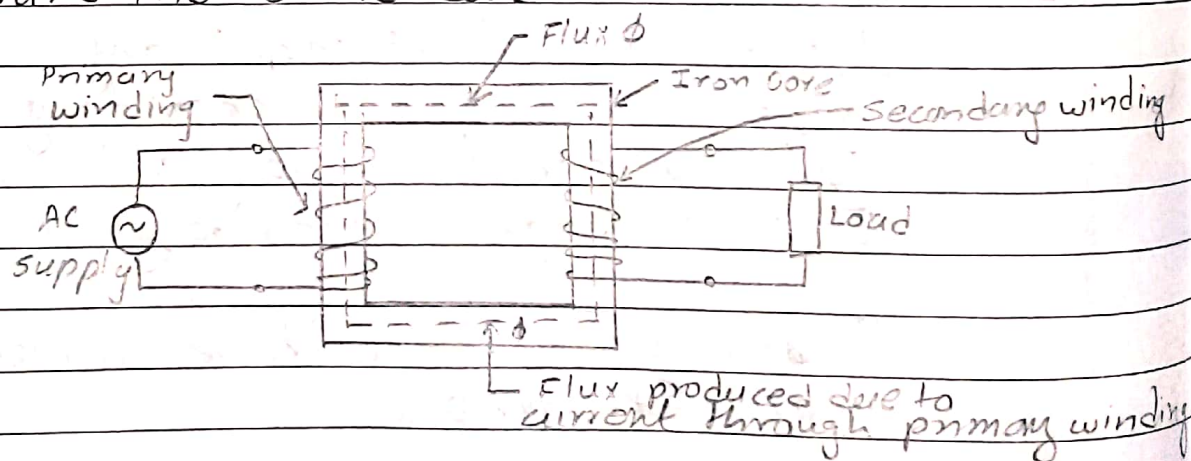
$$\frac{N_B}{N_A} = \frac{E_B}{E_A} = k = \text{voltage transformation ratio}$$

If $N_B > N_A$, $k > 1$, step up transformer
 $N_B < N_A$, $k < 1$, step down transformer

- Q Design a transformer. Explain the working principle and construction of a single phase AC transformer.
- Q Differentiate between step up and step down
- Q What do you understand by voltage transformation ratio

(Q1) Design a transformer. Explain the working principle and construction of a single phase AC transformer.

⇒ A transformer is a static electrical device that transfers electrical energy between two or more circuits. A varying current ~~is~~ in one coil of the transformer produces a varying magnetic flux, which in turn, induces a varying ~~ex~~ electromotive force across a second coil wound around the same core.



The working principle of the single phase transformer is based on the Faraday's law of electromagnetic induction. Basically, mutual induction between two or more windings is responsible for transformation action in an electrical transformer.

Transformer Construction:

The three ~~windings~~ parts of a transformer are:

- Primary Winding:

The winding that takes electrical power, and produces magnetic flux when it is connected to an electrical source.

- Magnetic Core:

This refers to the magnetic flux produced by the primary winding. The flux passes through a low reluctance path linked with secondary winding creating a closed magnetic circuit.

- Secondary Winding:

The winding that provides the desired output voltage due to normal induction in the transformer.

(Q2) Different between Step up transformer & Stepdown transformer.

Stepup transformer	Stepdown transformer
1) The output voltage of step-up transformer is more than the source voltage.	1) The output voltage of step-down transformer is less than the source voltage.
2) LV winding of transformer is the primary and HV winding is secondary.	2) The secondary voltage of step-down transformer is less than its primary voltage.
3) The number of turns in primary winding is less than the secondary winding.	3) The number of turns in primary winding is more than the secondary winding.
4) Primary current of transformer is more than the secondary current.	4) Secondary current is more than the primary current.

Q3) What do you understand by voltage transformation ratio?

⇒ Voltage transformation ratio is defined as the ratio of number of turns in the secondary winding to the number of turns in the primary winding. It is also defined as the ratio of voltage at the secondary terminal to the voltage at the primary terminals. It is defined as the

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} //$$

Note: A DC generator can be used as a DC motor without any constructional.

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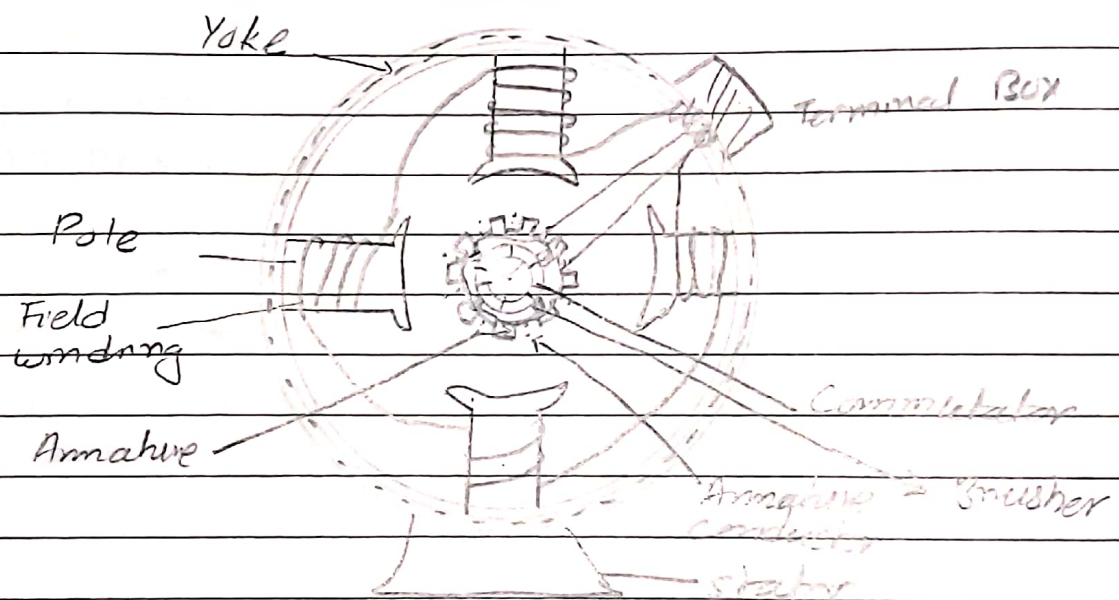
DC Generator and Motors

DC Generator:-

A DC generator is an electrical ^{machine} ~~machine~~ which converts mechanical energy into direct current electricity (D.C)

Construction:-

The following figure shows the construction of DC Generator.



4-pole DC machine (Generator/motor)

DC machine consist of two basic parts
① stator & ② rotor.

① Yoke:

The outer frame of a DC machine is called Yoke. It is made up of iron or steel. It provide mechanical support to the whole structure.

② Poles and Pole shoe:

Pole are joined to the yoke with the help of ~~to~~ bolts and ~~to~~ welding. They support field coils.

③ Field Winding:-

They are usually made of copper. They are ~~wound~~ set in such a way that when a energized ~~form~~ ~~a~~ form alternate north and south pole.

④ Armature Core (rotor):

It is cylindrical in shape with in laminated circular steel disk.

④ Commutator and brushes:

To keep the torque on a DC motor from reversing every time the coil moves through the plane perpendicular to the magnetic field, a split-ring device called a commutator is used to reverse the current at that point.

④ Armature Winding :-

It is usually a former wound copper coil which rests in armature slots.

④ Working Principle of DC Generator

According to the Faraday's law of electromagnetic induction, whenever a conductor is placed in a varying magnetic field (OR conductor is moved in a magnetic field), an emf gets induced in the conductor.

The magnitude of induced emf can be calculated from emf equation of DC Generator:

If the conductor is provided with the ~~#~~ closed path the induced current will circulate into the path.

In DC Generator, the field coils produce an ~~ex~~ electromagnetic field. And the armature conductor are rotated into the field. Therefore, the emf is generated. The direction of induced current is given by Fleming's right hand rule.

DC Motor:

The dc motor is a electrical device that converts electrical energy into mechanical energy.

The same dc machine can be used as motor or generator.

The construction is same of dc motor & dc generator.

Working principle of DC motor

The basic principle of dc motor is, "whenever a current carrying conductor is placed in a magnetic field, it experience mechanical force. The direction of this force is given by Fleming Left hand Rule and ~~magnitude~~ magnitude of it; $F = BIL$ where B = magnetic flux density
 I = current, L = length

When armature windings are connected to a dc supply, and electric current sets off in the windings. Magnetic field may be provided by using field winding or using permanent magnet. In this case current carrying armature conductor experience a force due to magnetic field. Commutator is made segmented to achieve unidirectional ~~for~~ torque (τ).

AC Motors

Single phase AC Motor

An AC motor is an electrical motor driven by an ~~etc~~ alternating current. The AC motor commonly consists of two basic parts, an outside stator having coil supplied with alternating current to produce ~~to~~ ^{rotating} magnetic field. And an inside rotor attached to the output shaft producing a secondary rotating magnetic field.

The single phase AC motor is driven by single phase AC current. Single phase AC motor can be classified into

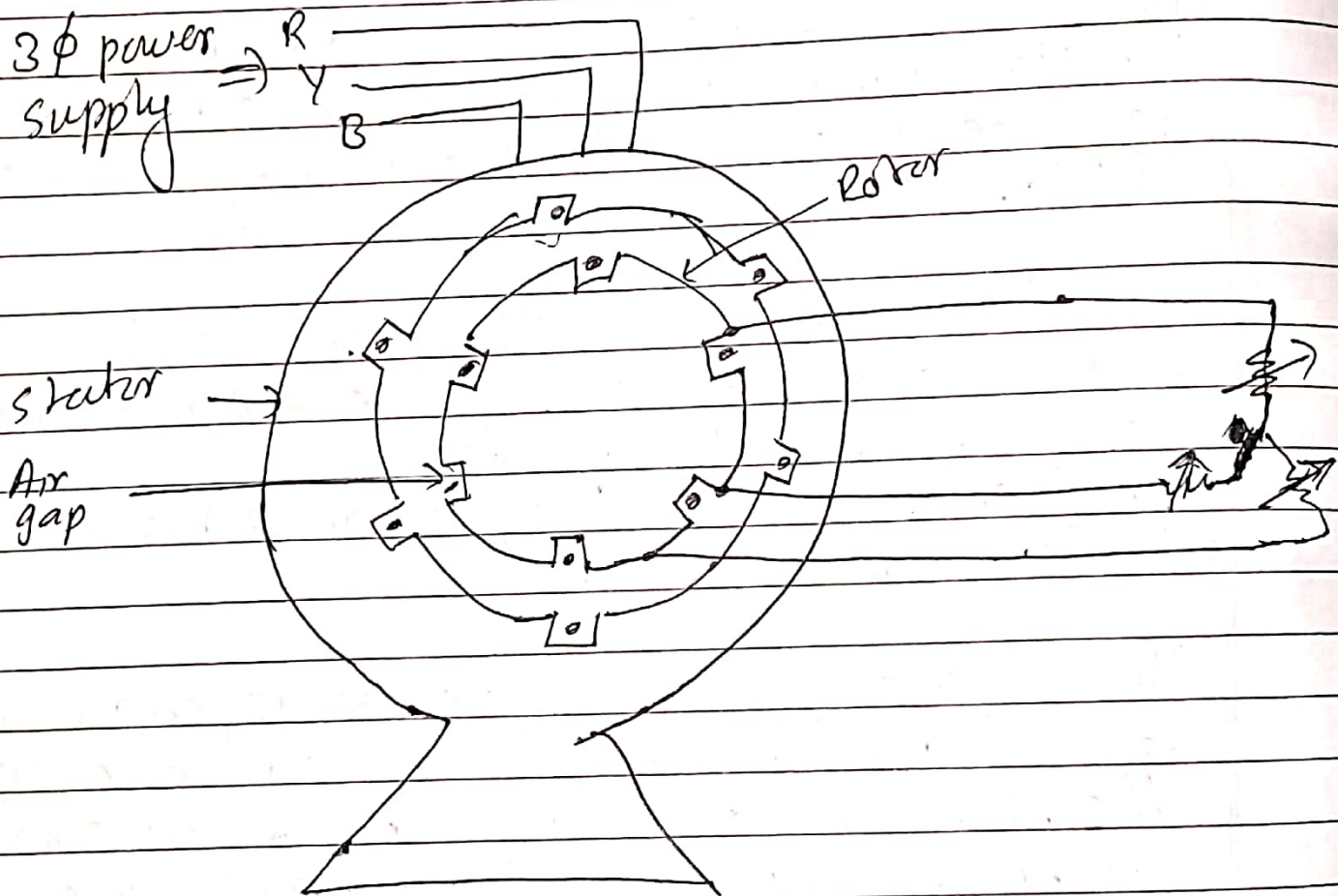
- ① induction motor (asynchronous motor)
- ② Synchronous motor

The induction motor always ~~re~~ depends on a small difference in speed between the stator rotating magnetic field and rotor shafts speed (slip) to induced rotor current in the rotor AC winding. So induction motor has low torque (T) than synchronous motor.

If synchronous motor there is no need for slip induction to operate. It uses either

permanent magnet or independently excited rotor winding. In single phase motor the stator is provided with single phase winding. The single phase motor are not self starting.

Three Phase induction Motor



3 phase induction motor

Construction of 3ϕ induction motor:-

The three phase induction motor is driven by 3 phase power supply. The stator has 3 phase winding distributed symmetrically on its inner periphery. This stator winding is energized by 3 phase power supply.

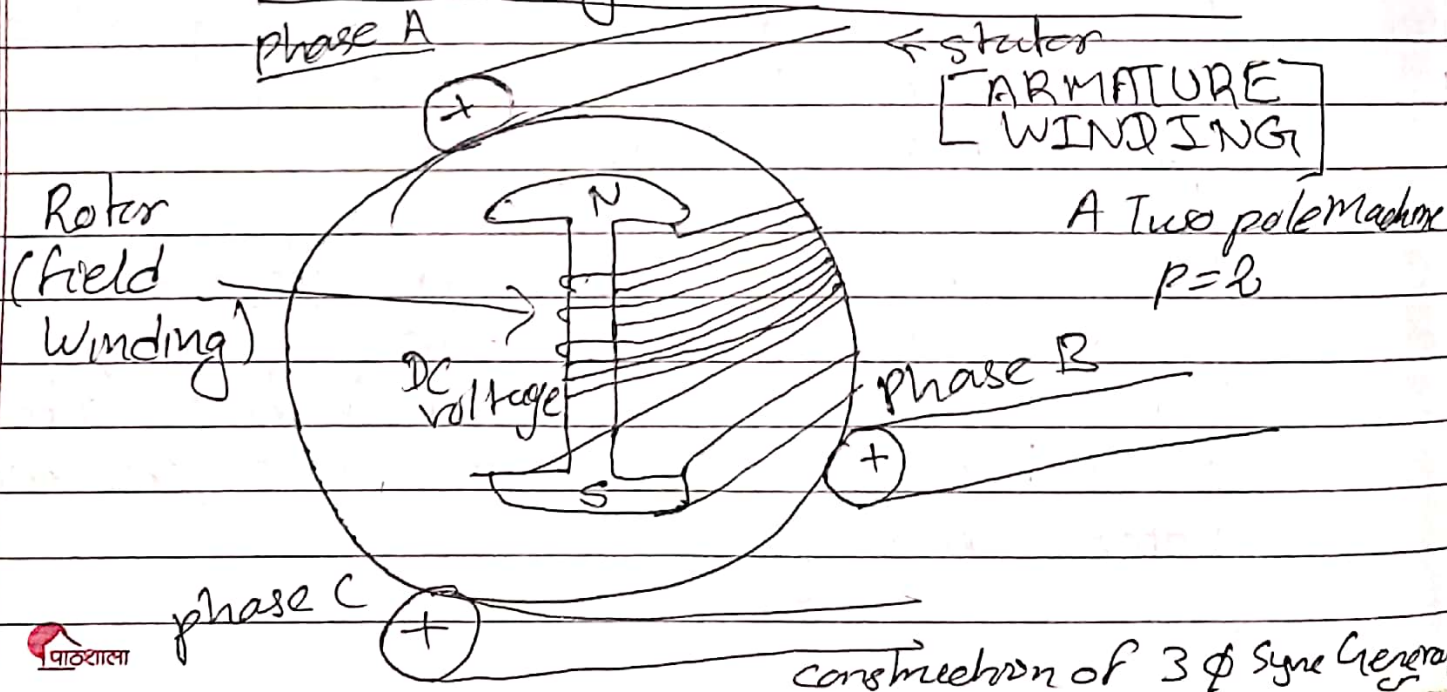
The rotor also has 3 phase winding on its periphery. But the rotor winding is not energized from any source and is short-circuited on itself.

Working principle:-

- When the 3 phase stator winding is energized from a 3 phase power supply, A rotating magnetic field is produced which rotates around stator at synchronous speed.
- The rotating magnetic field cuts the rotor conductors which were stationary. Due to this the emf is induced in the rotor conductors and current flows in the rotor winding.

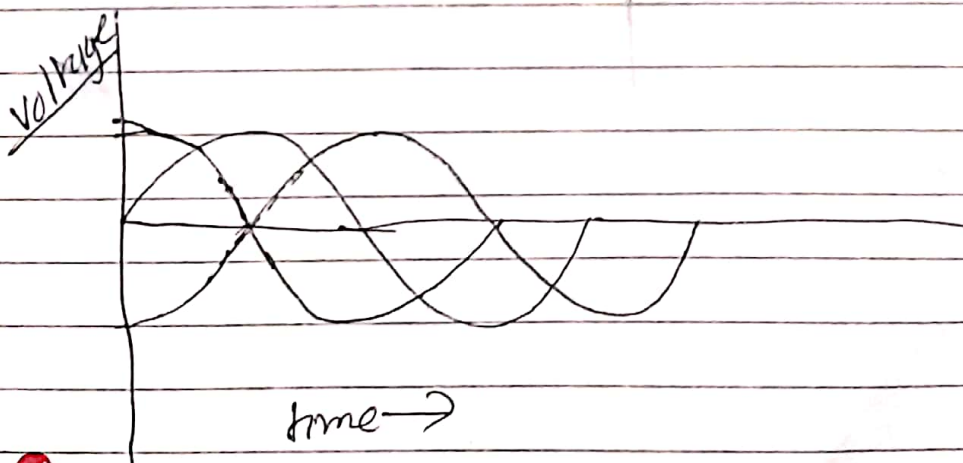
- Now, As per Lenz law, " the direction of the induced current will be such that it poses the very cause that produce it."
- Here, the cause of emf induction is the relative motion between the rotor conductor and stator's magnetic field. Hence, to reduce this, the rotor starts rotating in the same direction as that of stator field.
- to
- It ~~try~~ tries to caught the speed of stator winding speed. But can never caught speed due to friction and therefore, the motors keep rotating.

3-Phase Synchronous Generator



Working

The principle of operation of a ^{3 ϕ} synchronous generator is electromagnetic induction. If there exists a relative motion between the flux and conductors then an emf is induced in the conductor. Here, in a practical synchronous generator, the magnetic field rotates between the stationary armature conductors. The synchronous generator rotor and shaft are mechanically coupled to each other and rotates at synchronous speed. Thus, the magnetic flux cutting the armature conductors produces an induced emf which causes the current flow in armature conductors. For each winding, the current flows in one direction for the first half cycle and current flows in other direction for the second half cycle with the time lag of 120° . Hence, the output can be shown as figure below:-

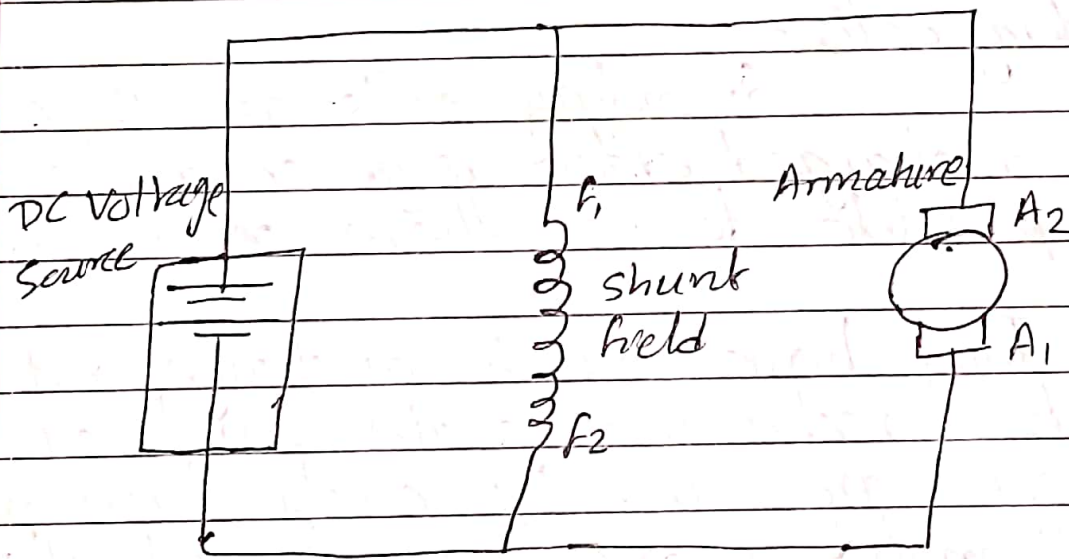


Q) Write the difference between induction motor & synchronous motor?

Synchronous Motor	Induction Motor
1) The construction of synchronous motor is comp difficult.	1) The construction of induction motor is easy.
2) Separate DC source is required for rotor excitation.	2) Rotor gets excited by induced emf so separate source is not necessary.
3) The speed is always synchronous irrespective of load.	3) The speed is always less synchronous.
4) Speed control is not possible.	4) Speed control is possible but difficult.
5) The motor is costly and requires frequent maintenance.	5) The motor is cheap and do not require frequent maintenance.

Shunt motor:-

A shunt motor (DC shunt motor) is a type of self excited DC motor. It is also known as shunt-wound motor. The construction of this motor is same as normal motor but the connections of field winding can be done parallel to the armature. This motor gives features like speed regulation, easy reversing control and produces low torque. Thus, this motor can be used for belt driven applications in industry application and automobiles.



Circuit diagram of shunt motor.

DC

— Prakash Paudel



Electrical & Electronics

Electronics is the branch of science that deals with the study of flow & control of electrons or electricity & the study of their behaviour & effects in vacuum gases & semi-conductors & with devices using such electrons.

Electrical devices take the energy of electrical current & transform it in simple ways into some other form of energy most likely light, heat or motion. Electronics devices take the electric energy & changes the shape & size. In contrast, electronic device do much more than electrical device.

The first electrical batteries were invented by scientist named Alessandro Volta in 1800.

The very first electronic device was invented in 1883 by Thomas Edison.

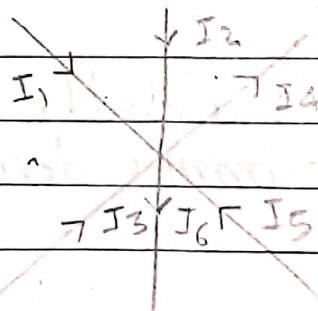
We can summarize few difference between electronics & electrical as below:

- * Electrical circuit do not have the ability to make decisions while electronic circuit have decision making ability.
- * Electrical mostly deals with the high voltage & high current while Electronics deals with low voltage & low current.

- * Electrical mostly deals with AC but not always & Electronic mostly deals with DC but not always.
- * Mostly electronic appliance uses 3 to 12 volt DC while electrical appliance uses 200 to 250 volt AC.
- * In computer electronic component are more than electronic component so it is called a electronic devices.
- * Electrical is about the production about of current while Electronic is about the controlling about of current & voltage.
- * Electronic is about study of diode, transistor, gate etc while Electrical is the study of generator, motors, hydropower.

KCL (Kirchoff's Current law)

It states that, "the algebraic sum of all the currents entering at the junction equal to the algebraic sum of all the currents leaving that junction."



In the above figure, using KCL we get;

incoming current = outgoing current
+ I_5

ie $I_1 + I_2 + I_3 = I_4 + I_6$

or, $I_1 + I_2 + I_3 + I_4 + I_5 - I_6 = 0$

KVL (Kirchoff's Voltage Law)

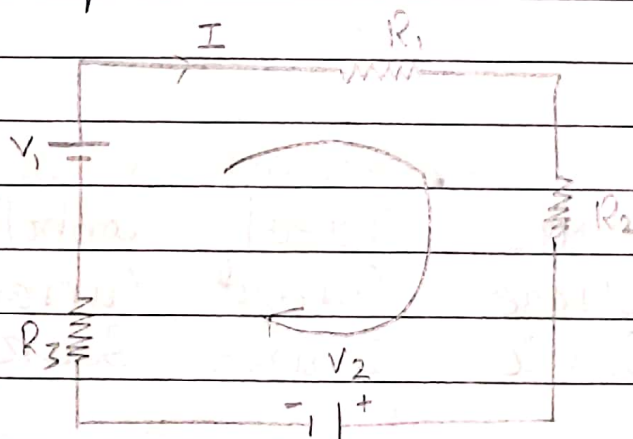
It states that, "the sum of voltages added around any closed loop in a circuit is equal to zero."

OR,

"The sum of emfs and products of current & resistance in a closed path in a circuit is always equal to zero."

ie. $\sum \text{emfs} + \sum IR = 0$

Example:



Here applying KVL we get;

$$V_1 - IR_1 - IR_2 - V_2 - IR_3 = 0$$

Energy & power

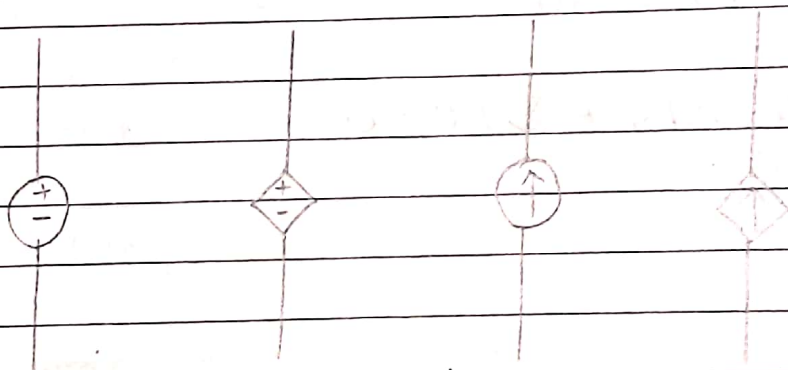
Energy is capacity to do work or energy is power integrated overtime. Its unit is Joule (volt-sec). Symbol of energy is W .

Power is rate at which work is done or power is rate at which energy is transmitted. Its unit is watt.

The relation between Energy & Power is given by the equation $P = \frac{W}{t}$.

where; t is the time.

Voltage Source & Current Source



Ideal
voltage
Source

Controlled
Voltage
Source

Ideal
Current
Source

Controlled
Current
Source

Most of the electrical energy are modeled as voltage sources.

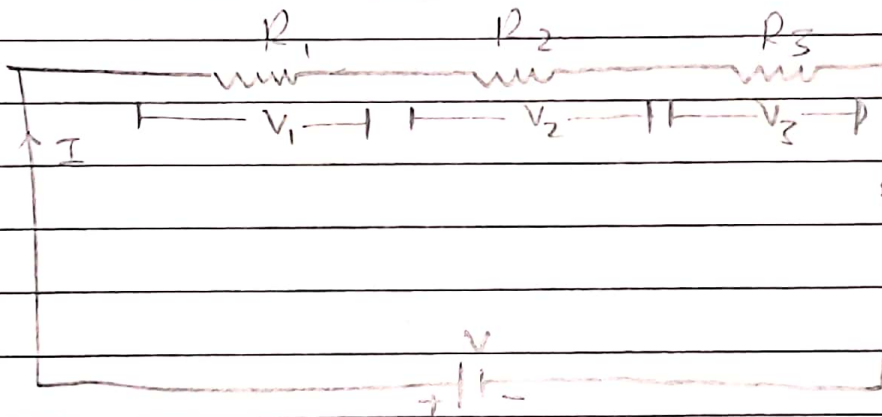
Series & Parallel Circuits

A series connection is a circuit in which components are arranged or connected in a chain.

A parallel connection is a connection in a circuit in which components are arranged with their heads connected together and their tail connected together.

S	P
C	V

① Resistance in Series



Equivalent resistances of resistors connected in Series $(R) = R_1 + R_2 + R_3 + \dots + R_n$

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$\text{or } IR = IR_1 + IR_2 + IR_3 + \dots + IR_n$$

$$\text{or } R = R_1 + R_2 + R_3 + \dots + R_n$$

Here;

$$V_1 = IR_1 = \frac{V}{R} \cdot R_1$$

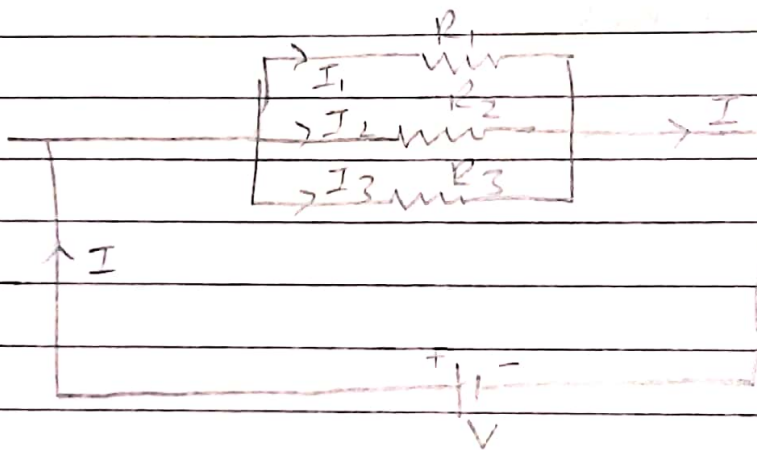
$$V_2 = IR_2 = \frac{V}{R} \cdot R_2$$

$$V_3 = IR_3 = \frac{V}{R} \cdot R_3$$

...

$$V_n = IR_n = \frac{V}{R} \cdot R_n$$

② Resistance in Parallel:-



The equivalent resistance of parallelly connected resistors is given by;

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

$$\& I = I_1 + I_2 + I_3 + \dots + I_n$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \dots + \frac{V}{R_n}$$

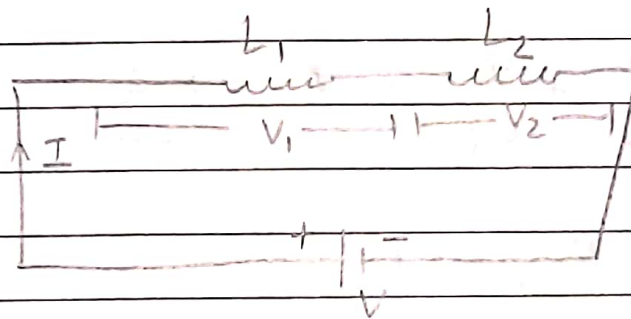
For two resistances R_1 and R_2 connected in parallel, the equivalent resistance is given by;

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{or, } \frac{1}{R} = \frac{R_2 + R_1}{R_1 \cdot R_2}$$

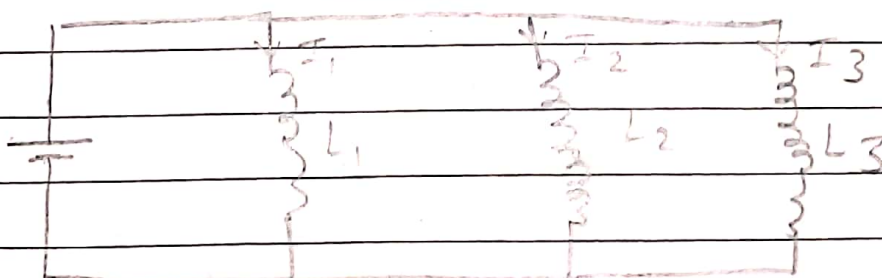
$$\text{or, } R = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

(*) Inductors in Series



$$L = L_1 + L_2 + L_3 + L_4 + \dots$$

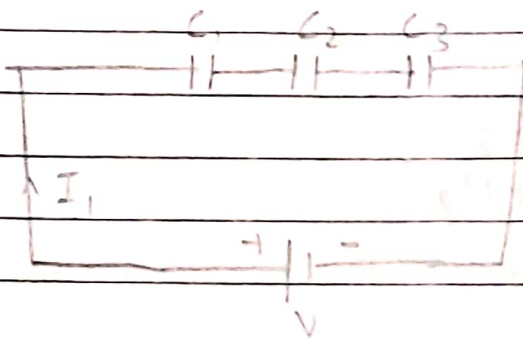
(*) Inductors in Parallel



$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

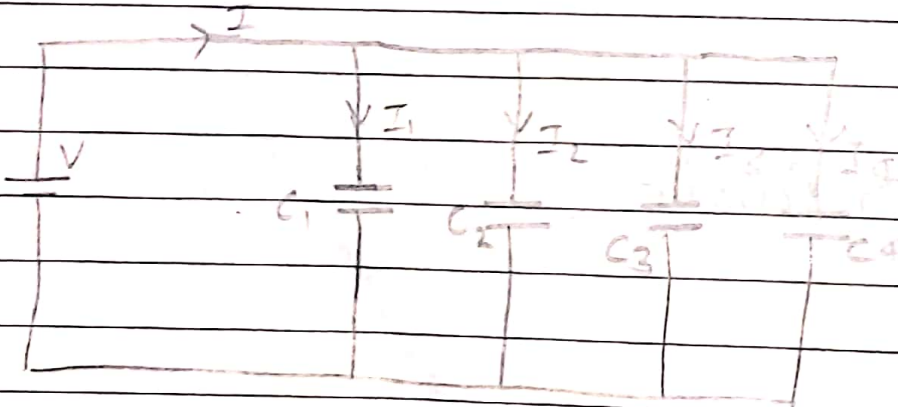
(*) Capacitance in Series:-



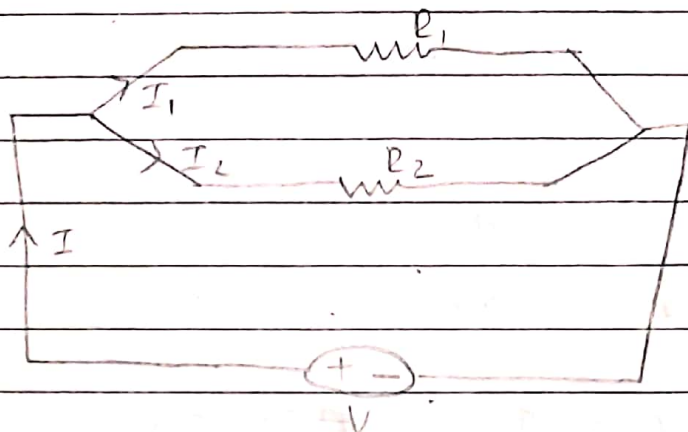
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

(*) Capacitance in Parallel

$$C = C_1 + C_2 + C_3 + \dots + C_n$$



Division of Current in parallel circuit



In the circuit given, Resistance R_1 & R_2 are connected in parallel. I_1 & I_2 are the currents flowing through R_1 & R_2 . Then using current division Rule we get;

$$I_1 = \frac{I \times R_2}{R_1 + R_2}$$

$$I_2 = \frac{I \times R_1}{R_1 + R_2}$$

Proof

$$I_1 = \frac{V}{R_1} \quad \text{--- (I)}$$

$$I_2 = \frac{V}{R_2} \quad \text{--- (II)}$$

$$\frac{I_1}{I_2} = \frac{V}{R_1} \times \frac{R_2}{V} = \frac{R_2}{R_1} \quad \text{--- (III)}$$

Also; we know that; $I = I_1 + I_2$
re $I_2 = I - I_1$ — (IV)

From eqn (III) & (IV)

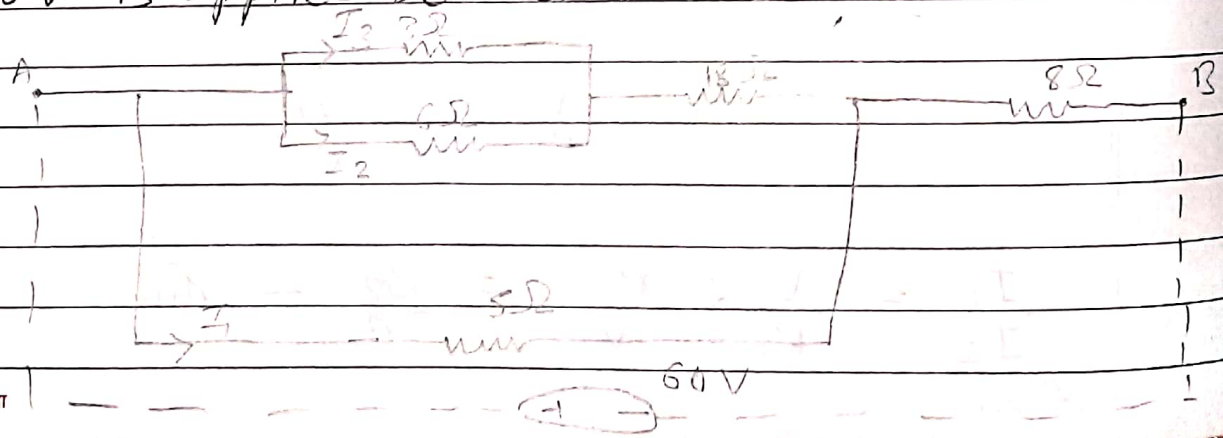
$$\frac{I_1}{I - I_1} = \frac{R_2}{R_1}$$

$$\begin{aligned} I_1 R_1 &= I R_2 - I_1 R_2 \\ I_1 R_1 + I_1 R_2 &= I R_2 \\ I_1 (R_1 + R_2) &= I R_2 \\ \therefore I_1 &= \frac{I R_2}{(R_1 + R_2)} \end{aligned}$$

Similarly;

$$I_2 = \frac{I \times R_1}{(R_1 + R_2)}$$

- ③ Calculate the effective resistance of the following combination of resistances & the voltage drop across each resistance when a potential difference of 60 V is applied between A and B.



$$R_T = (3/16 + 18) / 5 + 8$$

$$= 12$$

$$I_T = \frac{V_T}{R_T} = \frac{60V}{12} = 5A$$

$$\text{Voltage drop across } 8\Omega = I_T \times 8$$

$$= 5 \times 8$$

$$= 40V$$

$$\text{Voltage drop across AC} = (60 - 40) = 20V$$

$$\therefore V_S = 20V$$

Using current

$$I_2 = \frac{V_S}{5} = \frac{20}{5} = 4A$$

$$I_1 = 1A$$

$$I_3 = \frac{I_1 \times 6}{6+3} = \frac{1 \times 6}{9} = 0.667$$

$$I_4 = I_1 - I_3 = 1 - 0.667 = 0.33$$

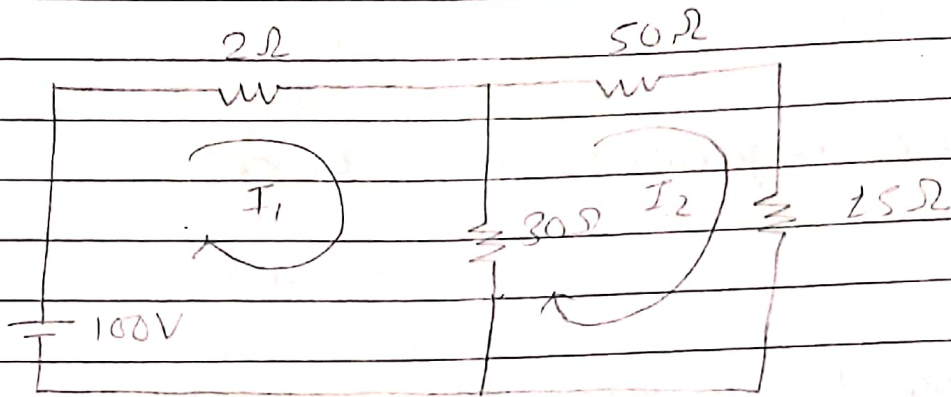
drop

$$\text{Voltage across } 3\Omega, V_3 = I_3 \times 3 = 0.667 \times 3 = 2V$$

$$V_6 = I_4 \times 6 = 0.33 \times 6 = 2V$$

$$V_{18} = I_1 \times 18 = 1 \times 18 = 18V$$

- ⑨ By mesh method, determined the total current drawn from the source & also current in 15Ω resistance of the given circuit. [10]



Solution;

Using mesh method;

Applying KVL at loop ①

$$100 - 2I_1 - 30(I_1 - I_2) = 0$$

$$\text{or, } 100 - 32I_1 + 30I_2 = 0$$

$$\text{or, } 32I_1 - 30I_2 = 100 \quad \text{--- ①}$$

Applying KVL at loop ② we get as;

$$\text{or, } -50I_2 - 15I_2 - 30(I_2 - I_1) = 0$$

$$\text{or, } 30I_1 - 95I_2 = 0 \quad \text{--- ②}$$

Calculate eqn ① & ② by and we get;

$$I_1 = 4.43 \text{ A}$$

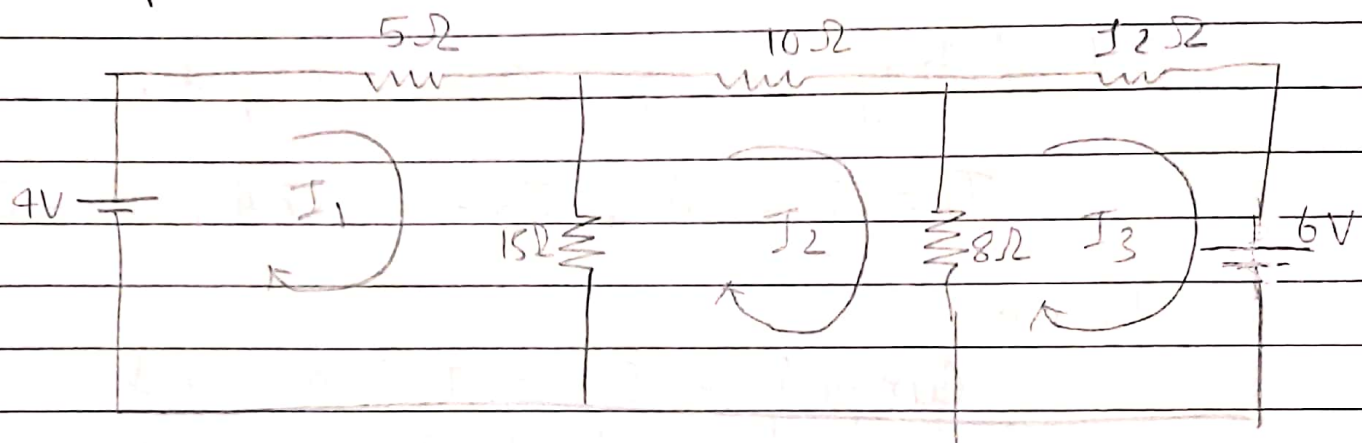
$$I_2 = 1.40 \text{ A}$$

∴ The current supplied by source = 4.43 A

&

The current flowing through $15\Omega = I_2 = 1.40\text{ A}$

- Q) A network is arranged as shown in the figure. Determine the value of currents in each resistor. (Using mesh method or maxwell loop method)



Solution;

Let I_1, I_2 & I_3 be the current in the closed loops as shown above.

Applying KVL at loop ① we get

$$\text{on } 4 - 5I_1 - 15(I_1 - I_2) = 0$$

$$\text{on } 4 - 5I_1 - 15I_1 + 15I_2 = 0$$

$$\text{on } -20I_1 + 15I_2 + 4 = 0$$

$$\text{on } 20I_1 - 15I_2 = 4 \quad \text{--- ①}$$

Applying KVL at loop ② we get;

$$-15(I_2 - I_1) - 10I_2 - 8(I_2 - I_3) = 0$$

$$-15I_2 + 15I_1 - 10I_2 - 8I_2 + 8I_3 = 0$$

$$15I_1 - 33I_2 + 8I_3 = 0 \quad \text{--- (II)}$$

Applying KVL at loop (3) we get;

$$-8(I_3 - I_2) - 12I_3 - 6 = 0$$

$$-20I_3 + 8I_2 - 6 = 0$$

$$8I_2 - 20I_3 = 6 \quad \text{--- (III)}$$

Calculating all eqn and we get;

$$I_1 = 0.224 \text{ A}, \quad I_2 = 0.032 \text{ A}, \quad I_3 = 0.29 \text{ A}$$

Again;

$$\text{Current in } 5\Omega = I_1 = 0.224 \text{ A}$$

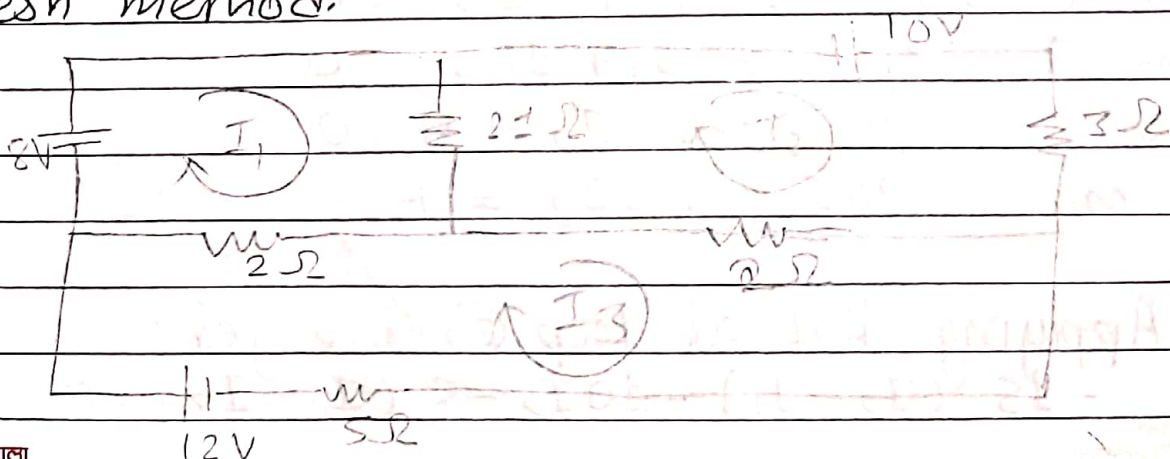
$$\text{Current in } 10\Omega = I_2 = 0.032 \text{ A}$$

$$\text{Current in } 12\Omega = I_3 = 0.29 \text{ A}$$

$$\text{Current in } 15\Omega = I_1 - I_2 = 0.192 \text{ A}$$

$$\text{Current in } 8\Omega = I_2 - I_3 = 0.514 \text{ A}$$

- Q. A network is arranged as shown in figure. Determine the value of current in 5Ω resistor using mesh method.



Let I_1 , I_2 & I_3 be the current in the closed loops as shown

Applying KVL in ① loop we get

$$8 - 21(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$8 - 21I_1 + 21I_2 - 2I_1 + 2I_3 = 0$$

$$23I_1 - 21I_2 - 2I_3 = 8 \quad \text{--- (i)}$$

Applying KVL in ② loop we get

$$-21(I_2 - I_1) + 10 - 3I_2 - 2(I_2 - I_3) = 0$$

$$21I_1 - 26I_2 + 2I_3 = -10 \quad \text{--- (ii)}$$

Applying KVL in loop ③ we get;

$$-2(I_3 - I_1) - 2(I_3 - I_2) - 5I_3 + 12 = 0$$

$$2I_1 + 2I_2 - 9I_3 = -12 \quad \text{--- (iii)}$$

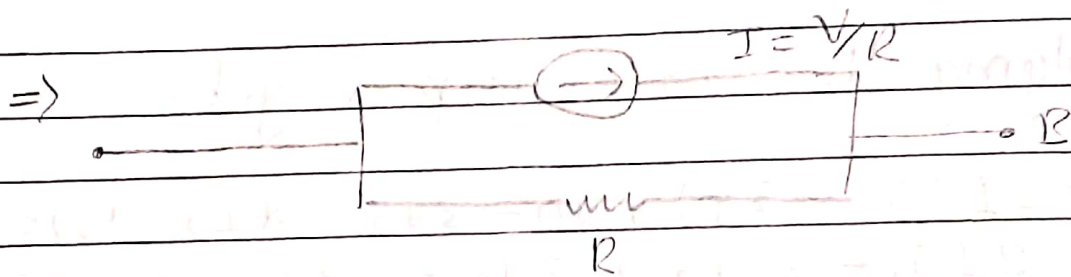
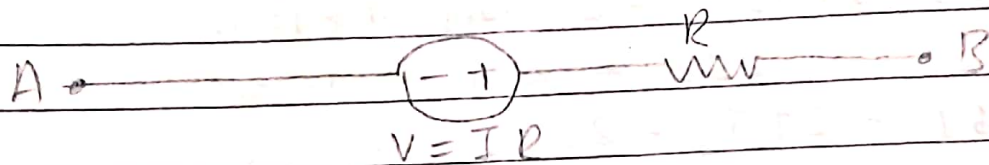
$$I_1 = 4.66 \text{ A}$$

$$I_2 = 4.413 \text{ A}$$

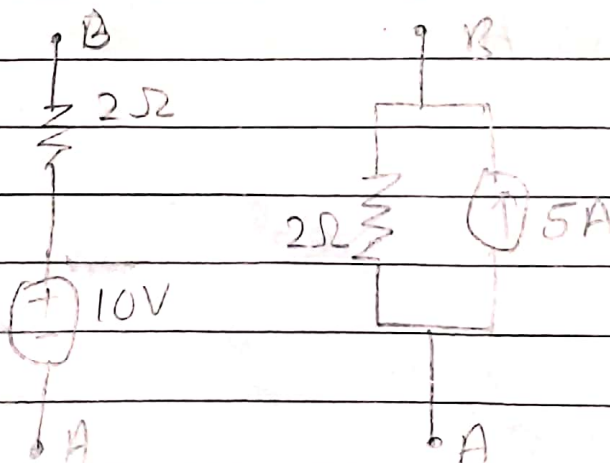
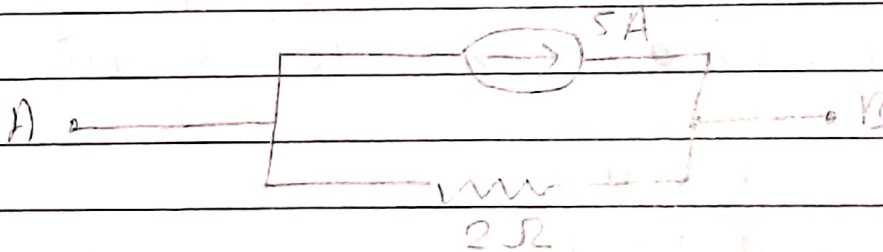
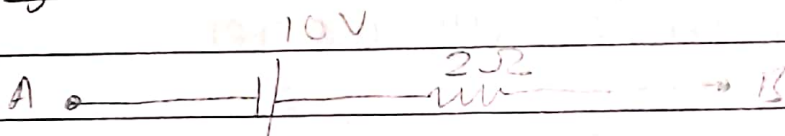
$$I_3 = 3.35 \text{ A}$$

* Voltage and Current Source Conversion

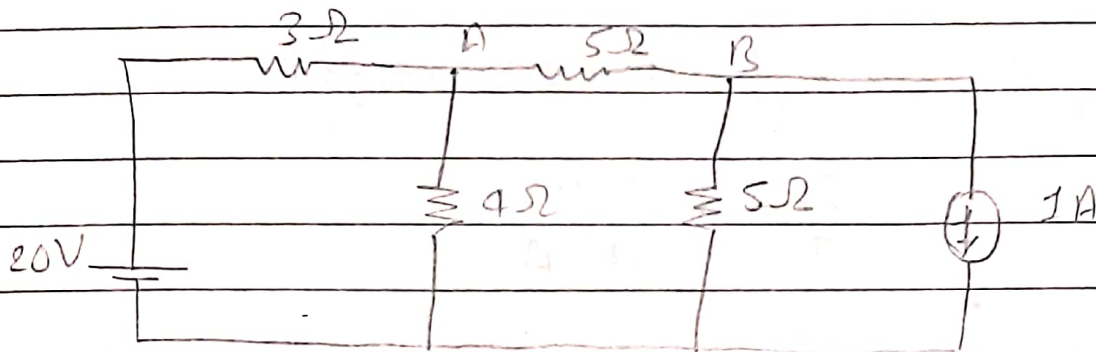
* A Voltage source with a resistance in series can be converted to a current source with a resistance in parallel & vice versa.



Eg:

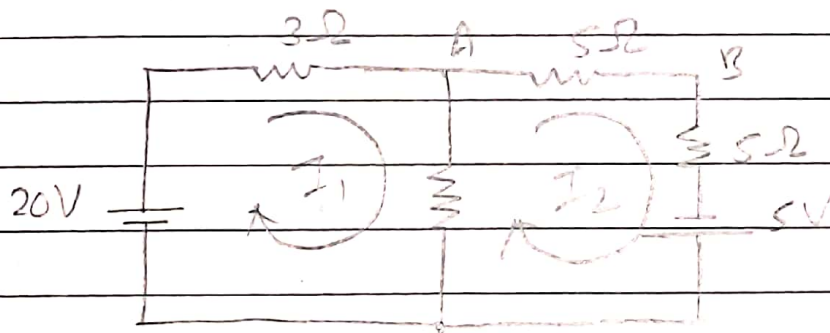


- ⑧ Evaluate the magnitude & direction of the current flowing through 5Ω resistor between points A & B of the circuit using maxwell-loop current method.



Solution

Converting $1A$ current source into voltage source & redrawing the circuit we get as follows.



Let I_1 & I_2 be the current flowing in loop as shown in fig.

Applying KVL at loop ①

$$20 - 3I_1 - 4(I_1 - I_2) = 0$$

$$7I_1 - 4I_2 = 20 \quad \text{--- ①}$$

Applying KVL at loop (1)

$$-4(I_2 - I_1) - 5I_2 - 5I_2 + 5 = 0$$

$$4I_1 - 14I_2 = -5 \quad \text{--- (1)}$$

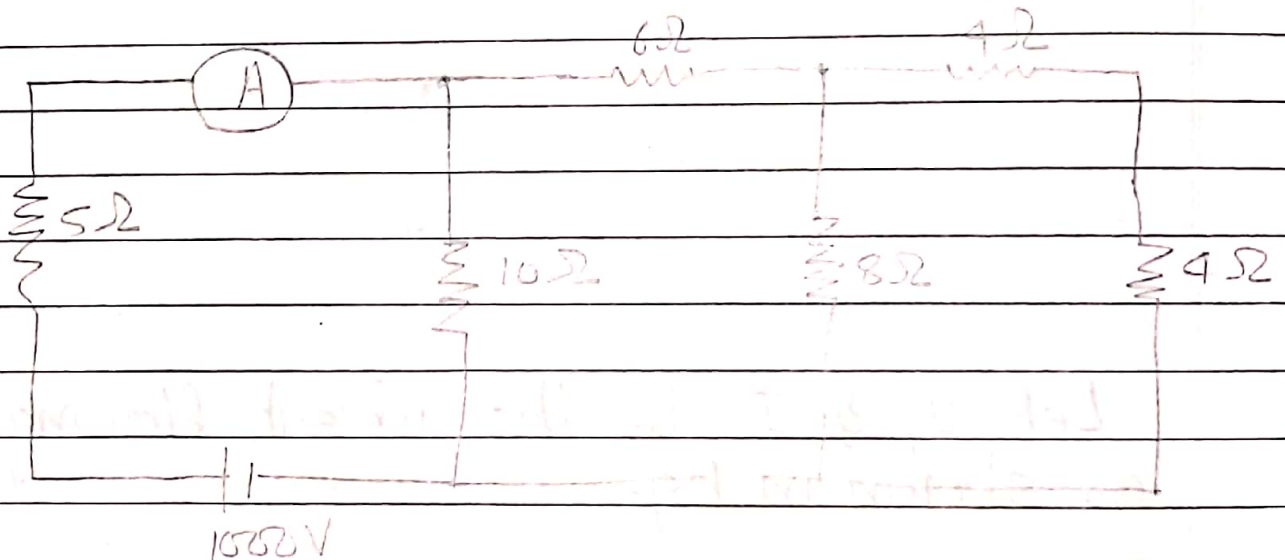
By calculating all eqn we get;

$$I_1 = 3.65 \text{ A}$$

$$I_2 = 1.40 \text{ A}$$

$\therefore$ The magnitude of current flowing through 5Ω resistor is 1.40 A & direction is from A to B.

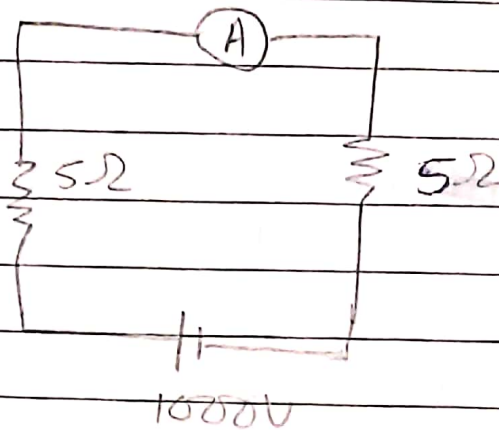
Q) What will be the value of Ammeter (A) in the figure below:



Solution,

4Ω & 4Ω are in series so we can add them which is 8Ω and both 8Ω are in parallel so

which makes 4Ω and 4Ω and 6Ω also in series so we add again and we get 10Ω then 10Ω and 10Ω are parallel so we make 5Ω . The result figure we get:



Again

$$\text{Total resistance} = 10\Omega = R$$

$$\text{Voltage } (V) = 1000V$$

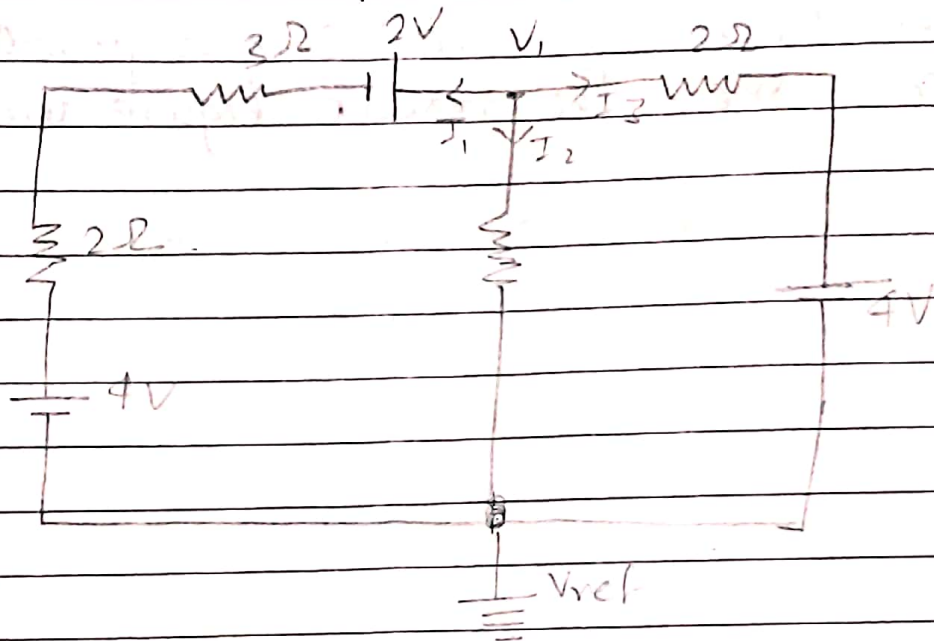
Ammeter is shown the value as follows;

$$I = \frac{V}{R}$$

$$= \frac{1000}{10}$$

$$= 100A$$

- Q Use Node voltage method, Find the current in 3Ω resistance for the network.



Solution:

Using Nodal method we get at node V_1 .

$$\frac{V_1 - 2 - 4}{3 + 2} + \frac{V_1}{2} + \frac{V_1 - 4}{2} = 0$$

$$\frac{V_1 - 6}{5} + \frac{V_1 + V_1 - 4}{2} = 0$$

$$\frac{V_1 - 6}{5} + \frac{2V_1 - 4}{2}$$

$$\frac{V_1 - 6}{5} + V_1 - 2 = 0$$

$$V_1 - 6 + 5V_1 - 10 = 0$$

$$6V_1 - 16 = 0$$

$$6V_1 = 16$$

$$V_1 = \frac{16}{6}$$

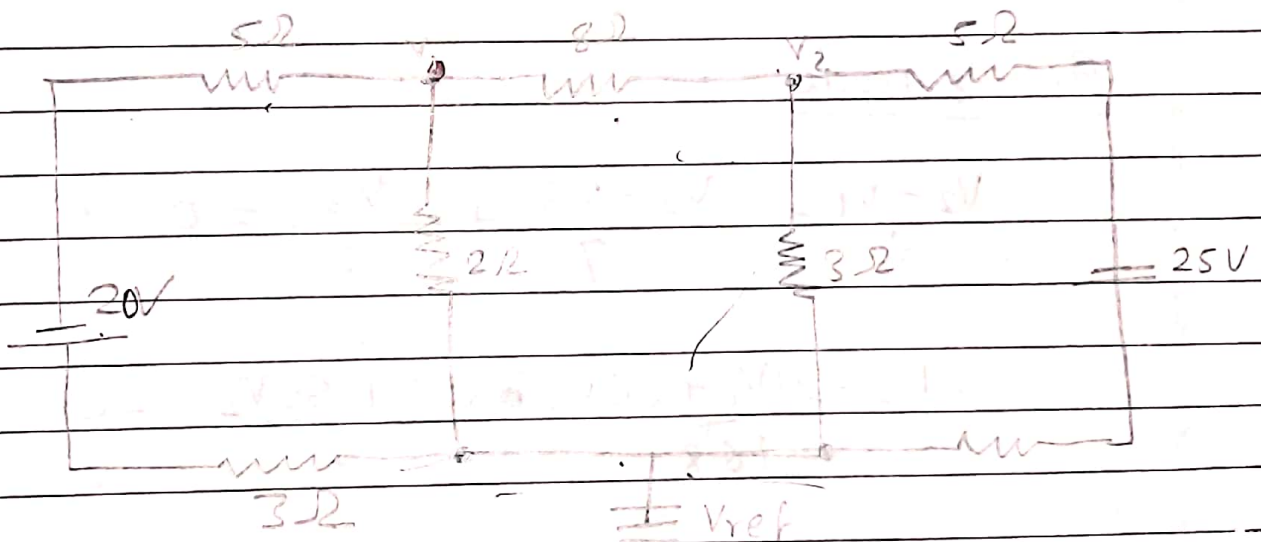
$$V_1 = 2.66 \text{ V}$$

in 3Ω resistance

$$\frac{V_1 - 2 - 4}{3 + 2} = -0.66 \text{ A}$$

i.e. 0.66 A is opposite direction.

- ③ Use node voltage method, Find the current in 8Ω in resistance for the network.



Solution:

Applying nodal method at V_1 and V_2 we get at first node.

$$\frac{V_1 + 20}{5 + 3} + \frac{V_1}{2} + \frac{V_1 - V_2}{8} = 0$$

$$\frac{V_1 + 20}{8} + \frac{V_1}{2} + \frac{V_1 - V_2}{8} = 0$$

$$\frac{2V_1 - V_2 + 20}{8} + \frac{V_1}{2} = 0$$

$$\frac{2V_1 - V_2 + 20}{8} + 4V_1 = 0$$

$$6V_1 - V_2 + 20 = 0$$

$$6V_1 - V_2 = -20 \quad \text{--- (1)}$$

At V_2 node

$$\frac{V_2 - V_1}{8} + \frac{V_2 - 25}{7} + \frac{V_2}{3} = 0$$

$$\frac{21V_2 - 21V_1 + 56V_2 - 600 + 56V_2}{168} = 0$$

$$21V_1 - 101V_2 = 600 \quad \text{--- (11)}$$

By calculating eqn (1) & (11) we get

$$V_1 = -4.47 \text{ V}$$

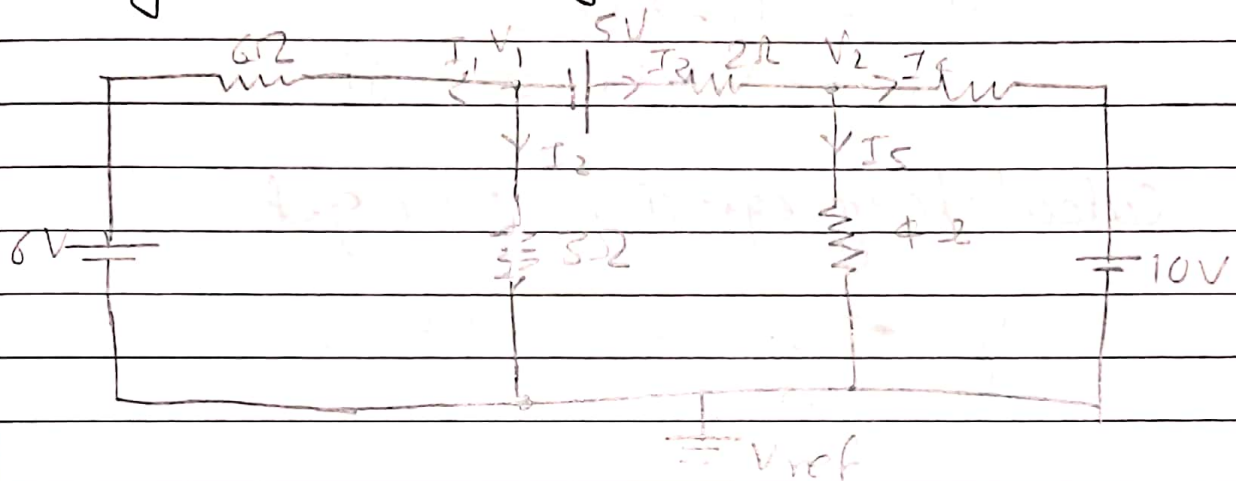
$$V_2 = -6.87 \text{ V}$$

In 8Ω resistance;

$$I = \frac{-4.47 + 6.87}{8}$$

$= -0.3A$ which is in opposite direction

④ Find the ~~the~~ branch currents in the given circuit using nodal analysis.



Let; V_1 & V_2 be the node at I_1, I_2, I_3 & I_4, I_5 are the current flowing through the circuit

At node V_1

$$\frac{V_1 - 6}{6} + \frac{V_1}{3} + \frac{V_1 + 5 - V_2}{2} = 0$$

$$\frac{V_1 - 6}{6} + 2V_1 + 3V_2 + 15 - 3V_2 = 0$$

$$6V_1 - 3V_2 = -9 \quad \text{--- ①}$$

At node V_2

$$\frac{V_2 - 10}{4} + \frac{V_2}{9} + \frac{V_2 - 5 - V_1}{2} = 0$$

$$\frac{V_2 - 10 + V_2 + 2V_2 - 10 - 2V_1}{4} = 0$$

$$4V_2 - 2V_1 - 20 = 0$$

$$2V_2 - V_1 - 10 = 0 \quad \text{--- (1)}$$

Calculating eqn ① & ② we get

$$V_1 = -5.33$$

$$V_2 = 7.66$$

By finding current we get;

$$I_1 = 0.77 \text{ A}$$

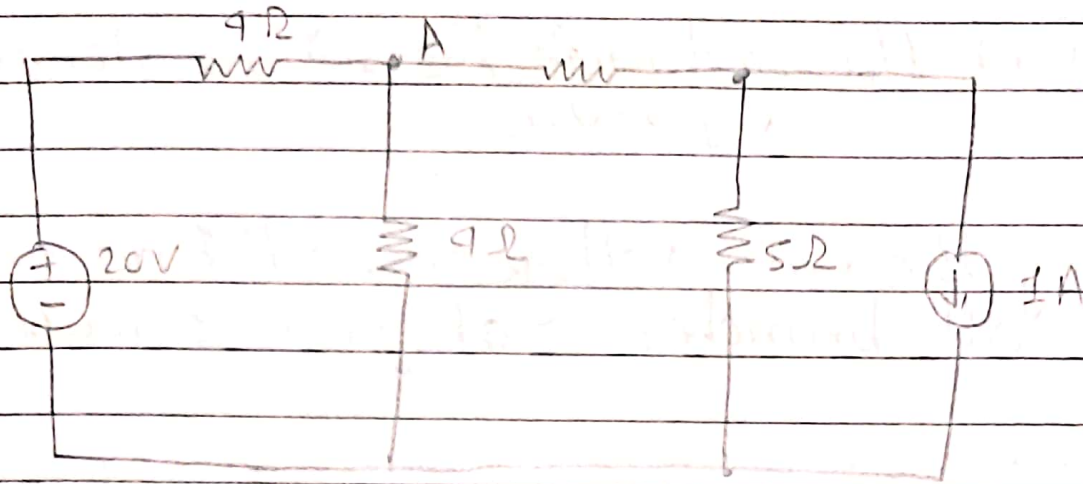
$$I_2 = 0.44 \text{ A}$$

$$I_3 = 0.335$$

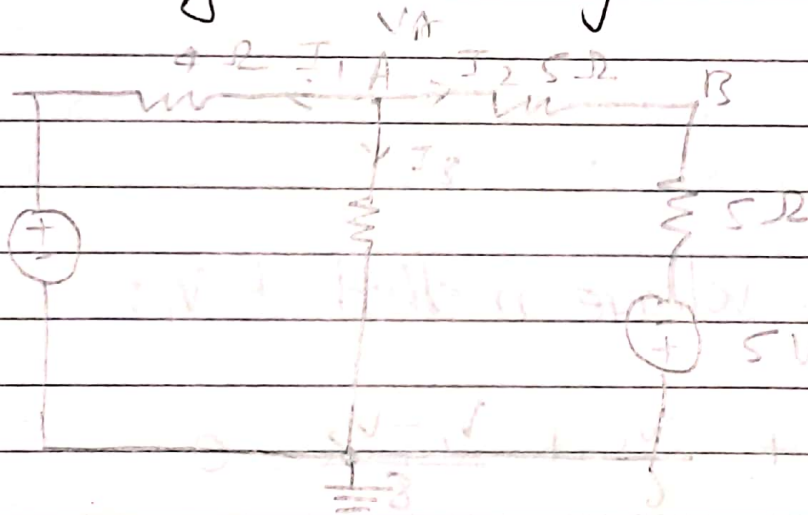
$$I_4 = 1.035 \text{ A}$$

$$I_5 = 1.415 \text{ A}$$

- Q Calculate the direction and magnitude of the current through the 5Ω resistor between A & B of the figure using nodal voltage method.



Converting $1A$ to voltage



$$\frac{V_A - 20}{4} + \frac{V_A + 5}{5+5} + \frac{V_A}{4} = 0$$

$$\frac{2V_A - 20}{4} + \frac{V_A + 5}{10} = 0$$

$$\frac{10V_A - 100 + 2V_A + 5}{20} = 0$$

$$12V_A - 90 = 0$$

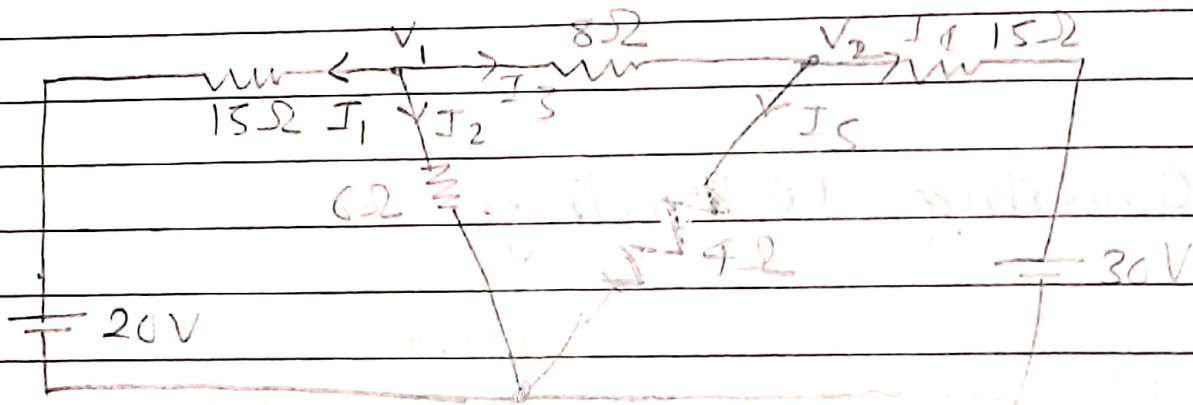
$$V_A = \frac{90}{12}$$

$$= 7.5V$$

$$\frac{7.5 + 5}{10} = 1.25A$$

Current flowing through the 5Ω resistor between A & B is $1.25A$.

- ⑨ Using the node voltage method. Find the current in all branches of given circuit below;



Using node voltage method at V_1 :

$$\frac{V_1 - 20}{15} + \frac{V_1}{6} + \frac{V_1 - V_2}{8} = 0$$

$$43V_1 - 15V_2 = 160 \quad \text{--- (1)}$$

Using node voltage method at V_2 :

$$\frac{V_2 - V_1}{8} + \frac{V_2}{4} + \frac{V_2 - 30}{15} = 0$$

$$-15V_1 + 53V_2 = 240 \text{ --- (11)}$$

Calculating eqn ① & ⑪ then we get;

$$V_1 = 5.88 \text{ V}$$

$$V_2 = 6.19 \text{ V}$$

Solving, Current are-

$$I_1 = 0.94 \text{ A}$$

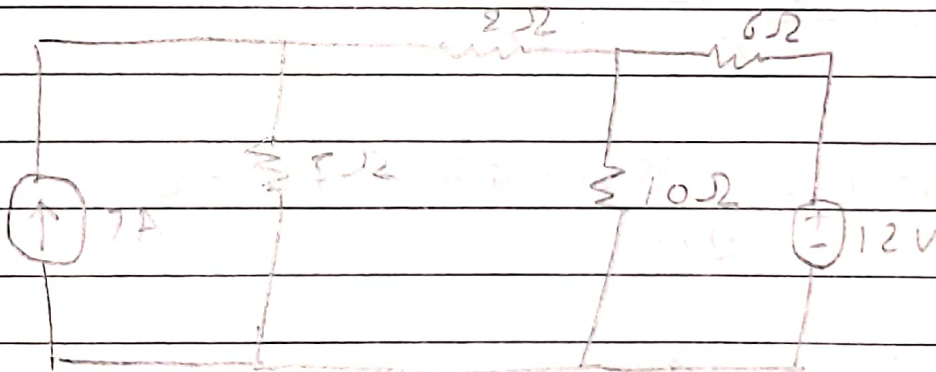
$$I_2 = 0.98 \text{ A}$$

$$I_3 = 0.038 \text{ A}$$

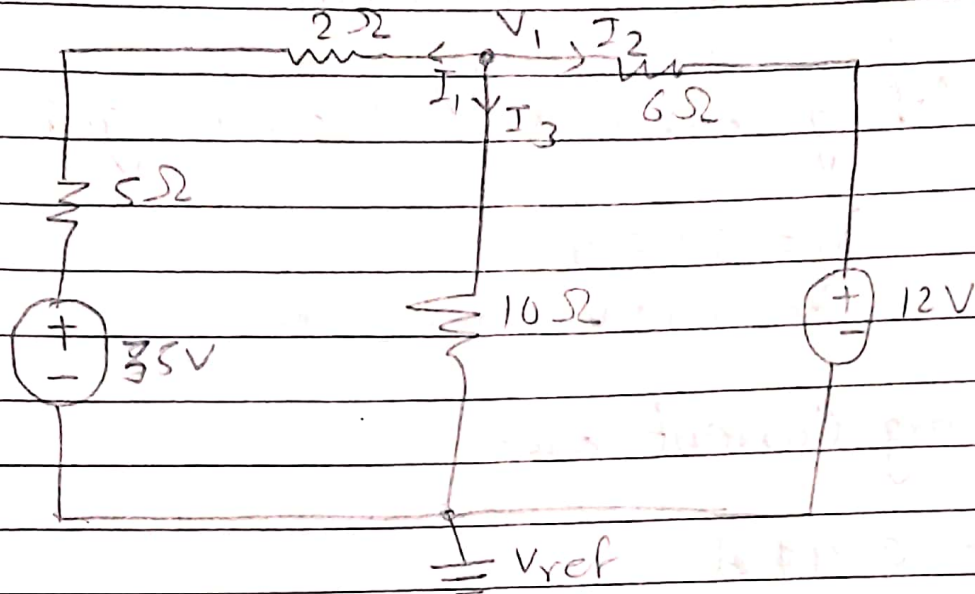
$$I_4 = 1.587 \text{ A}$$

$$I_5 = 1.54 \text{ A}$$

* Find the current flowing through 2Ω resistance from nodal analysis method.



Converting the 7A into the voltage & we get;



Assuming I_1 , I_2 & I_3 are the current flowing through the circuit:

Solving V_1 by using nodal analysis method;

$$\frac{V_1 - 35}{2 + 5} + \frac{V_1 - 12}{6} + \frac{V_1}{10} = 0$$

$$\frac{30V_1 - 1050 + 35V_1 - 420 + 21V_1}{210} = 0$$

$$86V_1 - 1470 = 0$$

$$V_1 = \frac{1470}{86}$$

$$= 17.09 \text{ V}$$

Now, Calculating the current flowing through 22 resistance

$$I_2 = \frac{V_1 - 35}{2 + 5}$$

$$= \frac{V_1 - 35}{7}$$

$$= \frac{17.09 - 35}{7}$$

$$= -2.55 \text{ A}$$

= 2.55 A where -ve sign indicates the current flowing in opposite direction.

Thevenin's Theorem:

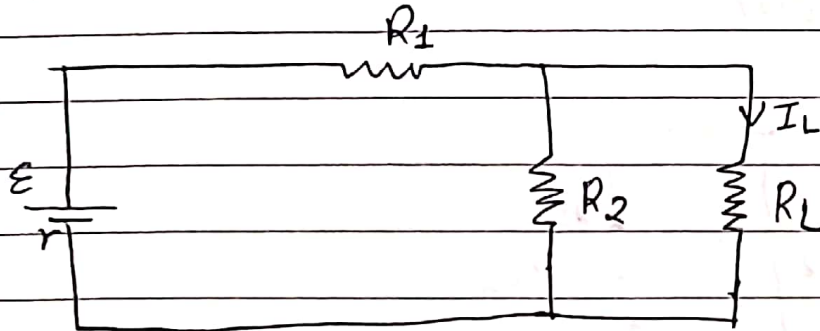
Thevenin's theorem states that, "The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active, bilateral network is given by;

$$I_L = \frac{V_{oc}}{R_{th} + R_L} = \frac{V_{th}}{R_{th} + R_L}$$

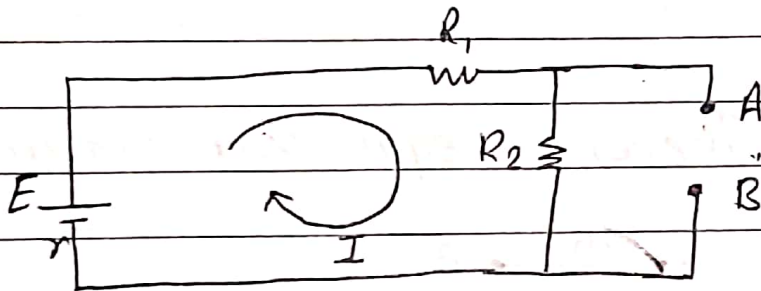
where; V_{oc} is the open circuit voltage (i.e. voltage across the two terminals when R_L is removed) and R_{th} is the internal resistance of the network as viewed ~~back~~ back into the open circuited network from terminals A and B with voltage source replaced by short circuit (leaving their internal resistance if any) and current source by open circuit (i.e. leaving by infinite resistance).

Procedure for Thevenizing the network

Suppose a network be like as following:



Step ① Remove the load resistance R_L , with open circuit terminals A & B. temporarily.



Step ②

Find the voltage $V_{oc} = V_{AB}$

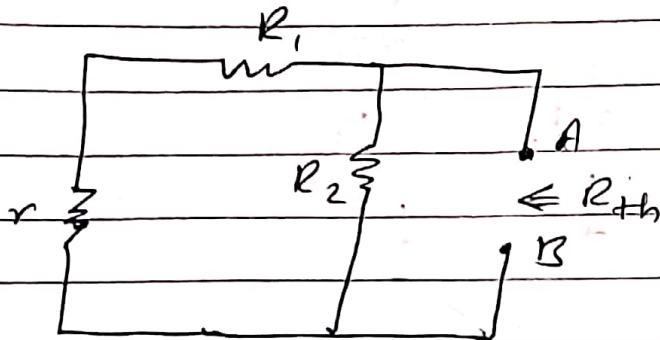
In the given circuit,

$$I = \frac{E}{(r + R_1 + R_2)}$$

$$\therefore V_{oc} = V_{R_2} = I \cdot R_2$$

Step 3:

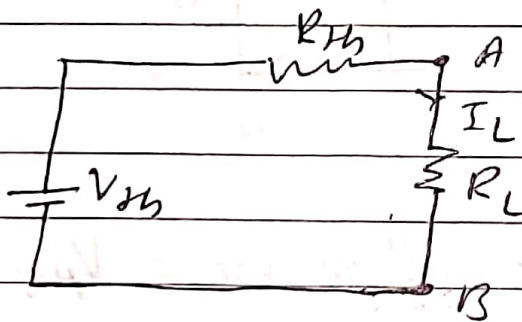
Find R_{th} by replacing voltage source by internal resistance if any & current source by open circuit.



$$R_{th} = R_{AB} = R_{oc} = (r + R_1) \parallel R_2$$

Step 4:

Draw Thevenins equivalent circuit as:



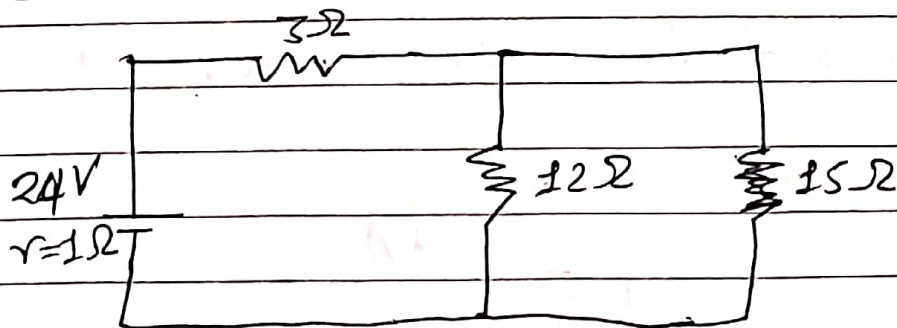
Step 5:

Apply the Thevenins formula to find I_L .

i.e.

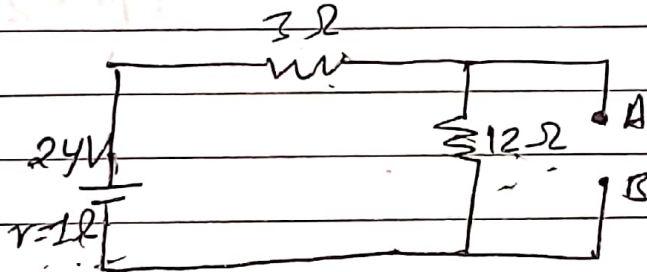
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

(Q) Apply the Thevenin's theorem to find the current flowing through 15Ω resistance.



Here; 15Ω is the R_L .

Step 1: Removing the 15Ω load resistance with terminal A B we get as;



Step 2: Calculating V_{oc}

$$I = \frac{V}{(r + R_1 + R_2)}$$

$$= \frac{24}{1 + 3 + 12}$$

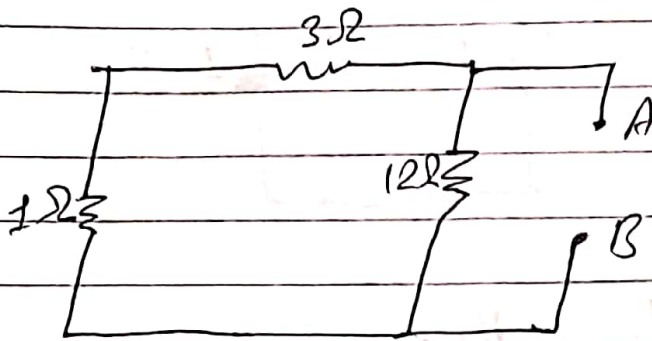
$$= \frac{24 \times 3}{16 \times 2} = 1.5 \text{ A}$$

$$\begin{aligned} V_{oc} &= I \cdot R_2 \\ &= 1.5 \times 12 \\ &= 18 \text{ V} \end{aligned}$$

Step III

Calculating R_{th}

We should draw the circuit as;

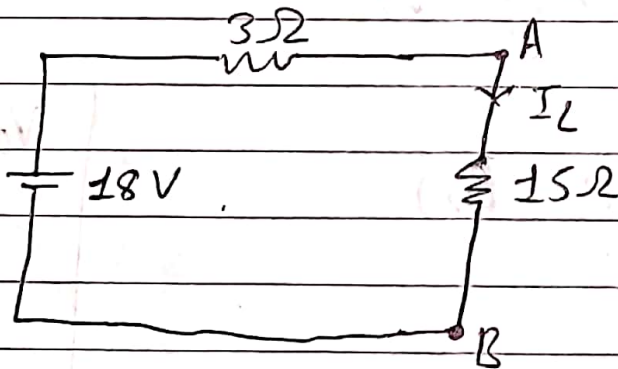


$$\begin{aligned} R_{th} &= (1 + 3) \parallel 12 \\ &= 4 \parallel 12 \\ &= \frac{48}{16} \\ &= 3 \Omega \end{aligned}$$

$$\frac{R_1 \cdot R_2}{R_1 + R_2}$$

Step IV:

The thevenins equivalent circuit is;



Step V: Applying the thevenin's formula to find I_L

$$I_L = \frac{18}{3 + 15}$$

$$= 1 \text{ A}$$

Hence, the current flowing through 15Ω resistance is 1 A .

Norton's Theorem:

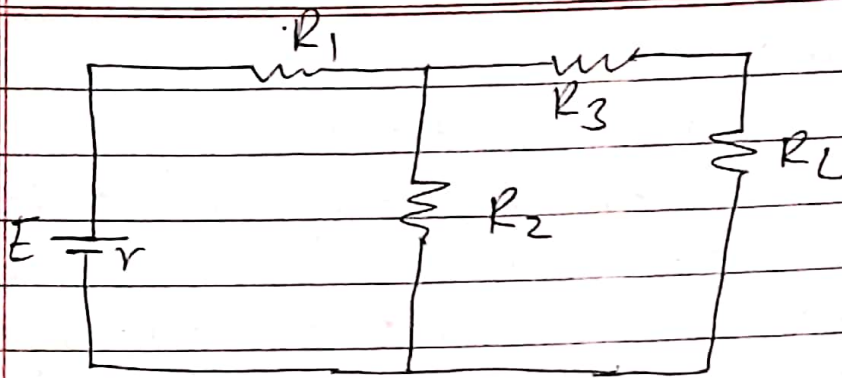
It states that, "Any two ~~elements~~ terminals of the network containing a linear active element can be replaced by an equivalent constant current source (I_N) with a parallel resistance (R_N) where; I_N is the current through a short circuit placed across of terminals A & B. R_N is the equivalent resistance of the network as seen from terminal A and B (open circuit) after all voltage source have been removed by their internal resistance & current source by infinite resistance that is;

The current flowing through load resistance is given by;

$$I_L = \frac{I_N \times R_N}{(R_L + R_N)}$$

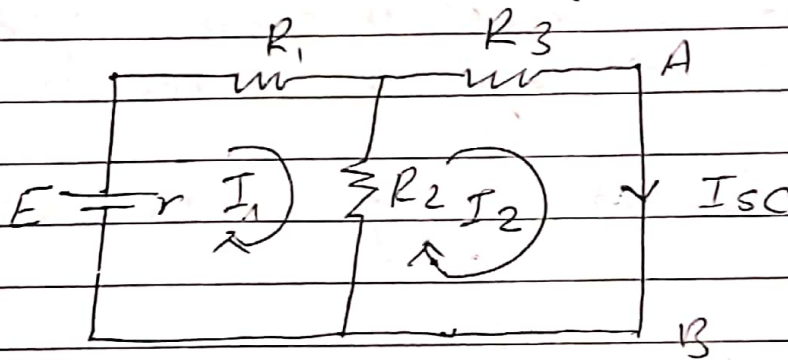
Procedure for Nortonizing Network:-

Suppose a Network as;
where r is the internal resistance of the ~~system~~ source.



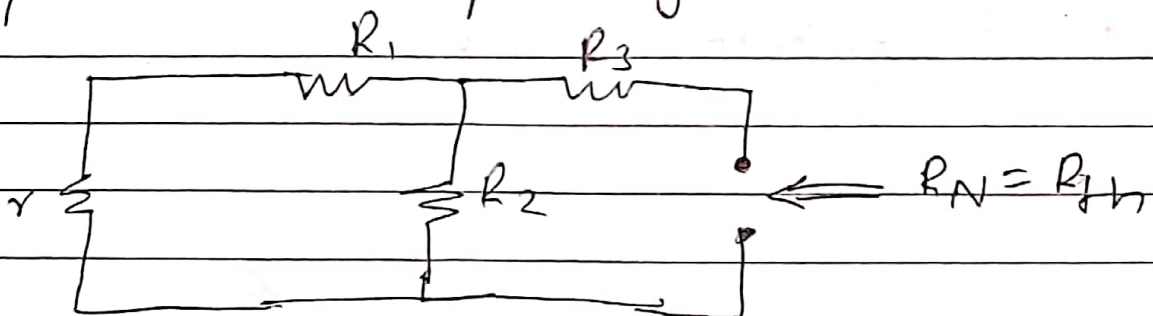
Step ①: Remove the load resistance by short circuit with terminals A and B.

Calculate $I_{sc} = I_N$ (step II)



Here; $I_{sc} = I_2$

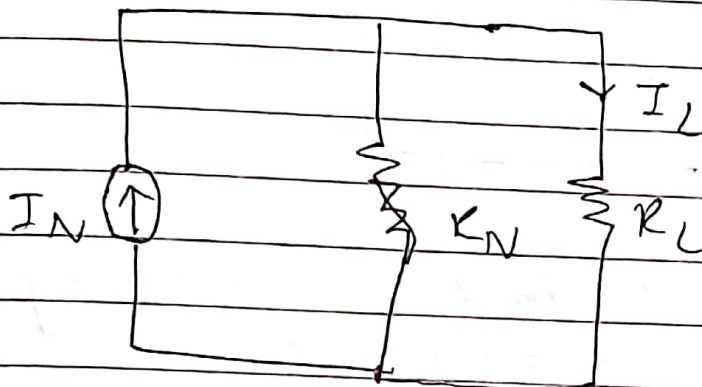
Step ③: Calculate $R_N = R_{th}$ by removing voltage source & current source with short & open circuit respectively.



Here;

$$R_{th} = [(r + R_1) \parallel R_2] + R_3$$

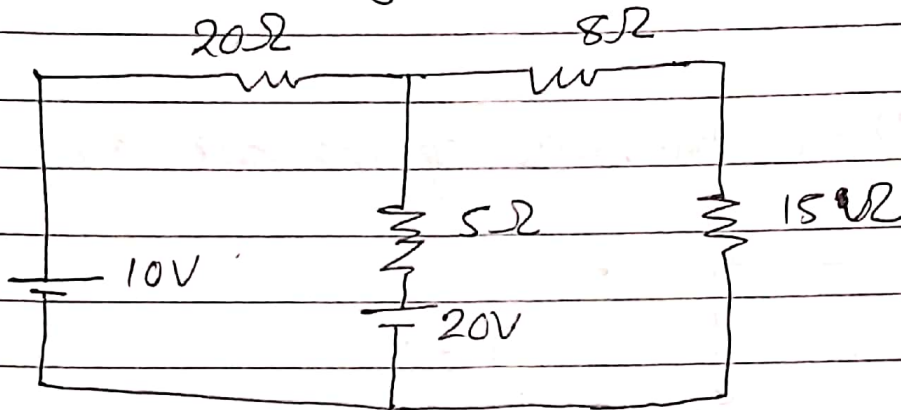
Step ④: Draw Norton's equivalent circuit as;



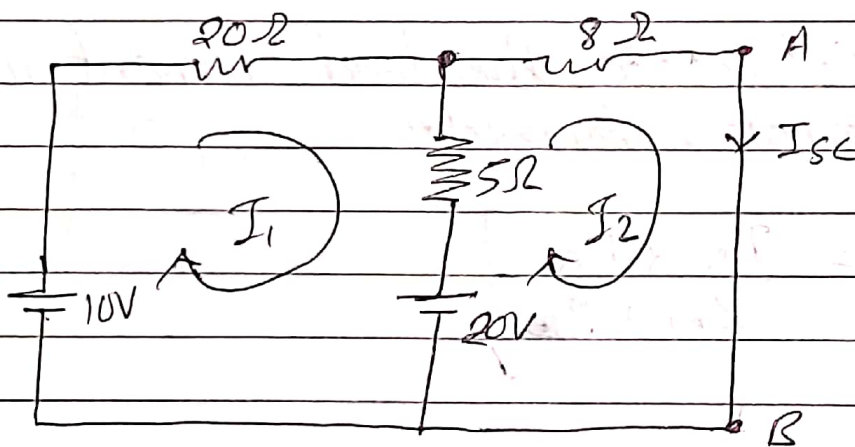
Step ⑤: Calculate the load current using Norton's formula;

$$I_L = \frac{I_N \times R_N}{R_N + R_L}$$

Q Determine the circuit flowing through 15Ω resistance using Norton's theorem.



step ①: Removing the load resistance by short circuit with terminals A & B.



step ② Calculating $I_{sc} = I_N$

$$\frac{V_1 - 10}{20} + \frac{V_1 - 20}{5} + \frac{V_1}{8} = 0$$

$$24V_1 - 20 + 8V_1 - 160 + 5V_1 = 0$$

$$13V_1 - 180 = 0$$

$$\frac{180}{25}$$

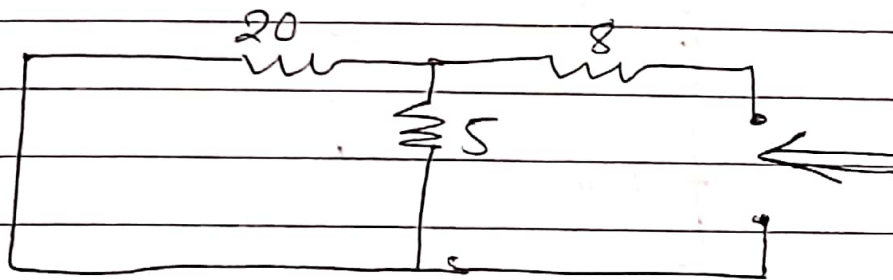
$$17V_1 = 180$$

$$V_1 = \frac{180}{17}$$

$$V_1 = \frac{12}{10.588} \text{ V}$$

$$\frac{V_1}{8} = \frac{12}{10.588} = 1.5 \text{ A}$$

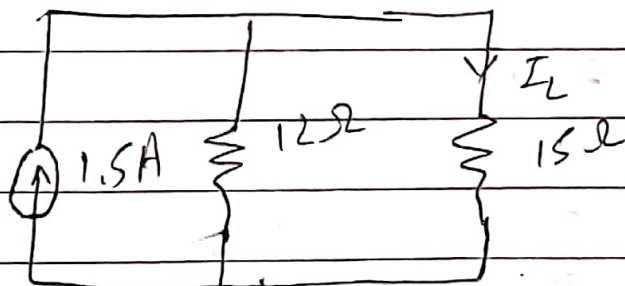
step (III) $R_{th} = R_N$



$$R_N = (20 // 5) + 8$$

$$= 12 \Omega$$

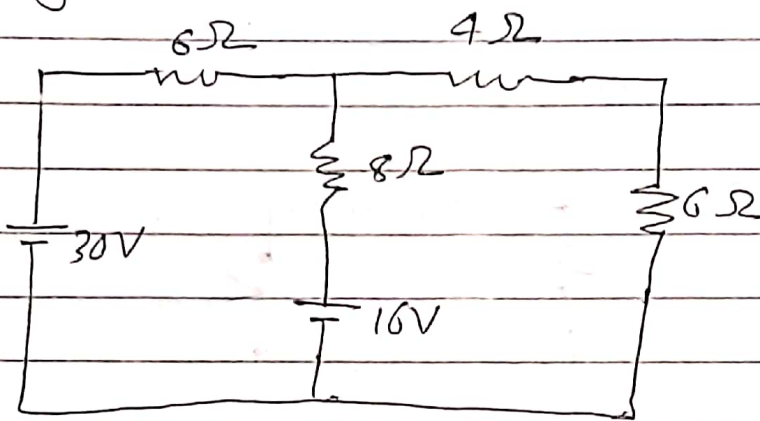
step (IV): Draw Norton's equivalent circuit



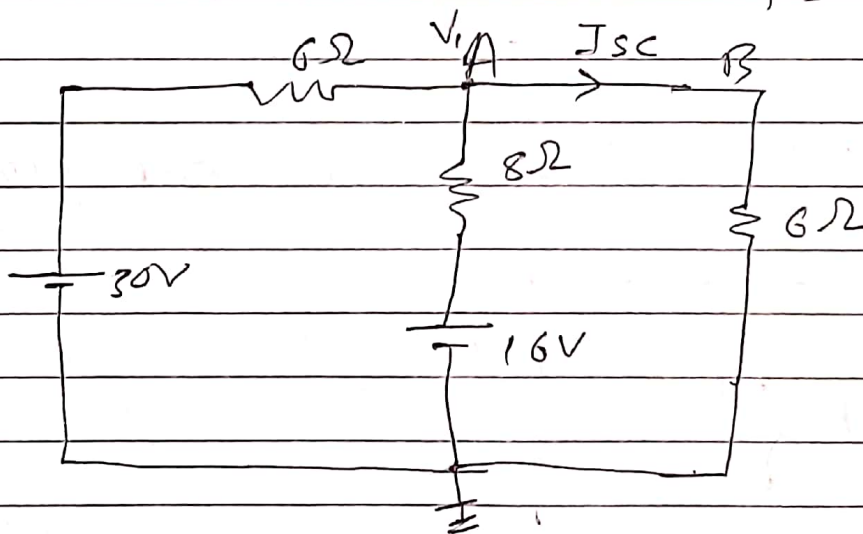
Step (v): Using Norton's formula;

$$I_L = \frac{1.5 \times 12}{12 + 15} = \frac{18}{27} = 0.67 \text{ A}$$

Q) Find the current flowing through 4Ω resistor in a given network using ~~norton's~~ norton's analysis.



Step (i): Removing the load resistance by short circuit with terminals A & B



$$2 \frac{6,8}{3,4}$$

Date: _____
Page: _____

step ④ Calculating $I_N = I_{sc}$

$$\frac{V_1 - 30}{6} + \frac{V_1 - 16}{8} + \frac{V_1}{6} = 0$$

$$\frac{2V_1 - 30}{6} + \frac{V_1 - 16}{8} = 0$$

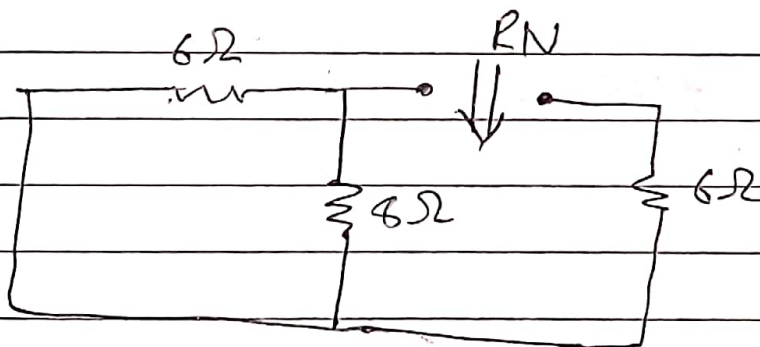
$$\frac{8V_1 - 120 + 3V_1 - 48}{24} = 0$$

$$11V_1 - 168 = 0$$

$$V_1 = \frac{168}{11} = 15.27$$

$$I_{sc} = \frac{V_1}{6} = \frac{15.27}{6} = 2.54 \text{ A}$$

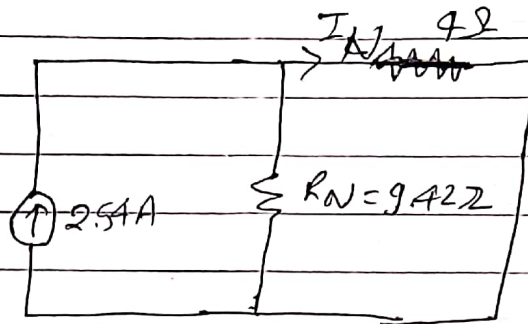
step ⑤ R_N



$$R_N = (6 \parallel 6) + 6$$

$$R_N = 9.42 \Omega$$

step (iv) Draw the Norton's equivalent circuit



step (v): Calculate I_N

$$I_{NL} = \frac{I_N \times R_N}{R_N + R_L}$$

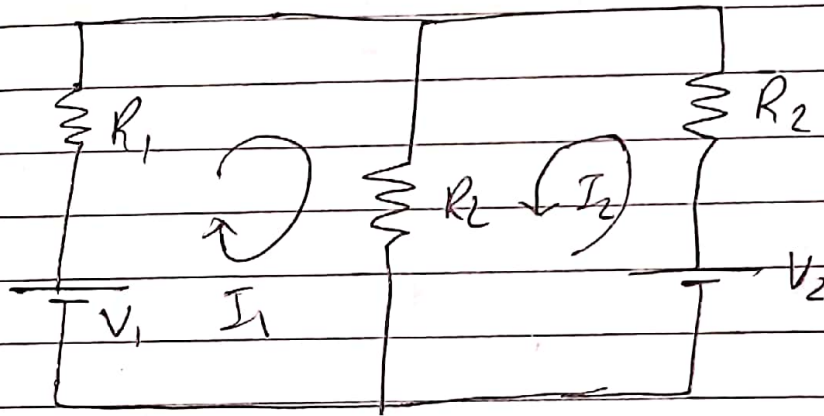
$$= \frac{2.54 \times 9.42}{9.42 + 4}$$

$$= 1.78 \text{ A}$$

Superposition Theorem:

In a network of linear resistances containing more than one generator (e.m.f.), the current which flows at any point is sum of all the current which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistance.

Ex:



In above circuit;

let I_1 = current through R_2 due to ~~source~~ source V_1 only.

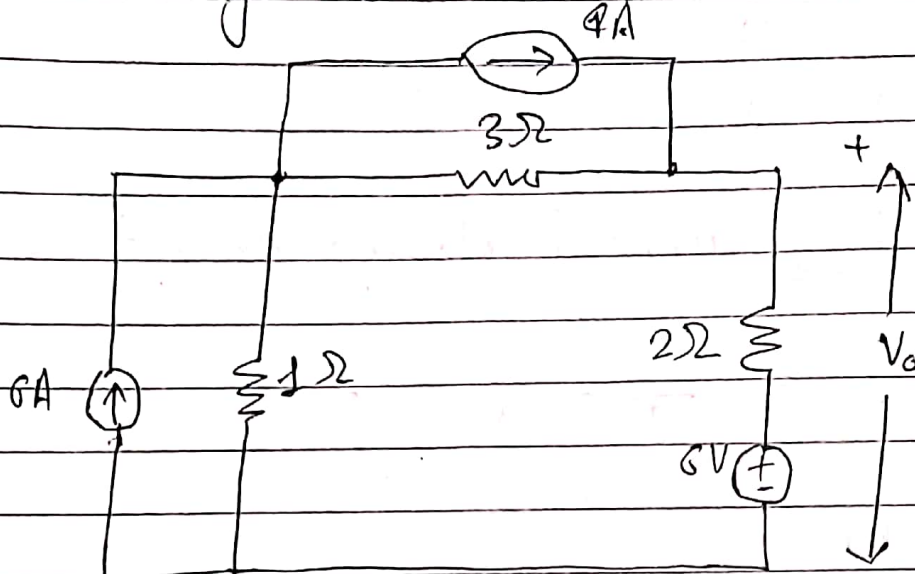
I_2 = current through R_2 due to source V_2 only.

$\therefore$ Total current flowing through $R_2 = I_1 + I_2$

$$V = IR$$

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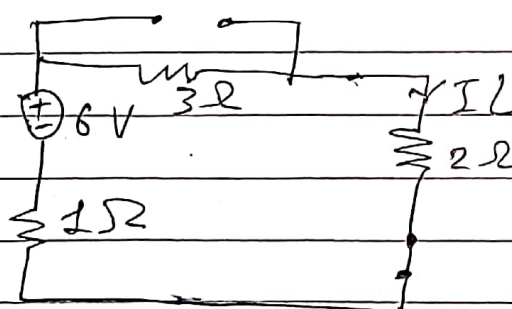
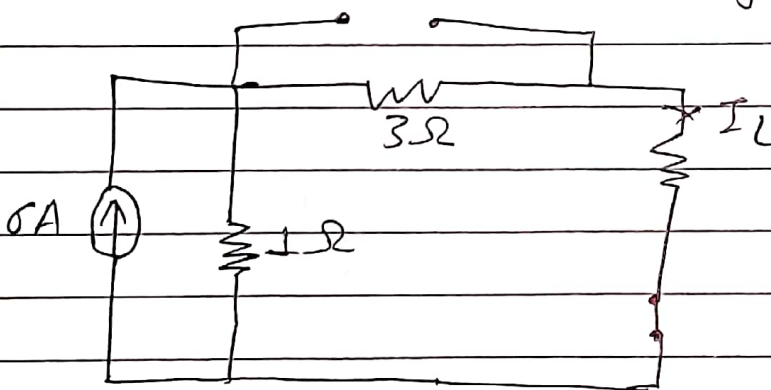
Q Using Superposition Theorem, find the value of output voltage V_o in the circuit



Solution:

Case I) Taking 6A ~~source~~ source only.

Redrawn the circuit we get;



~~I_L~~ ~~V_o~~
 ~~I_L~~ $\therefore \frac{6}{1+3+2} = 1$

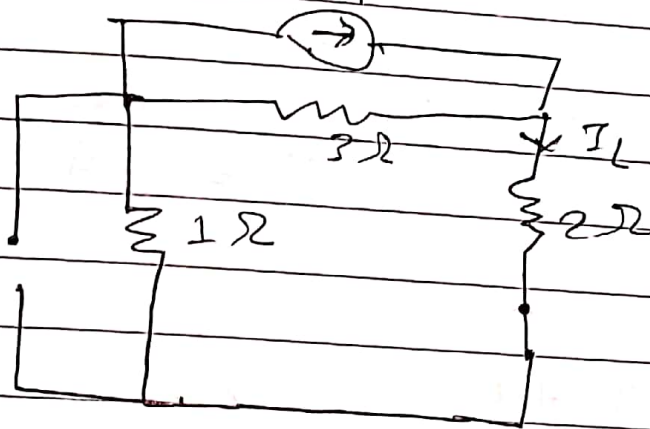
Using current division rule

$$I_L = \frac{6 \times 1}{(1+2+3)} = 1 \text{ A}$$

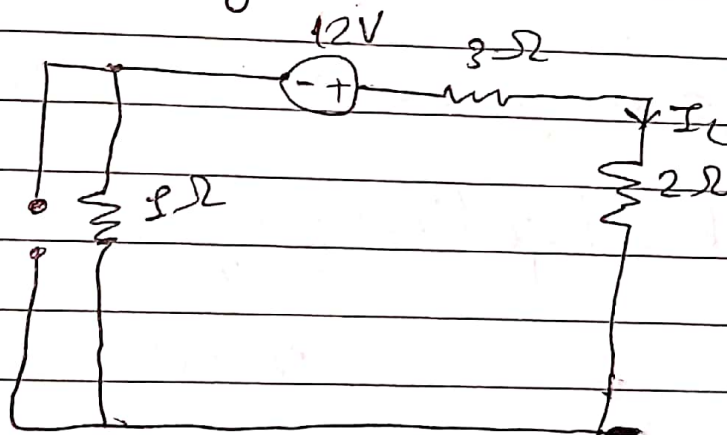
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~~Voltage drop across 2Ω~~

∴ Current through 2Ω by 6A (I_1) = 1A
taking 4A source only

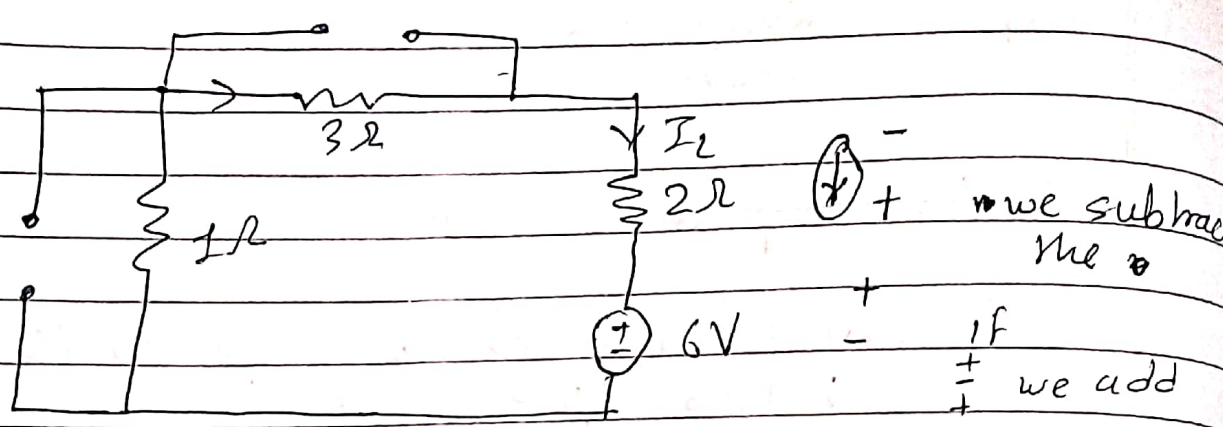


Converting



$$\frac{12}{3+2+1} = 2 \text{ A}$$

∴ Current through 2Ω by 4A (I_2) = 2A



Using KVL

$$-I_1 - 3I_2 - 2I_3 - 6 = 0$$

$$I_3 = -1 \text{ A}$$

Now;

$$I = I_1 + I_2 + I_3$$

$$= 1 + 2 - 1$$

$$= 2 \text{ A}$$

$\therefore$ Voltage drop across $2\Omega = I \times 2 = 2 \times 2 = 4 \text{ V}$

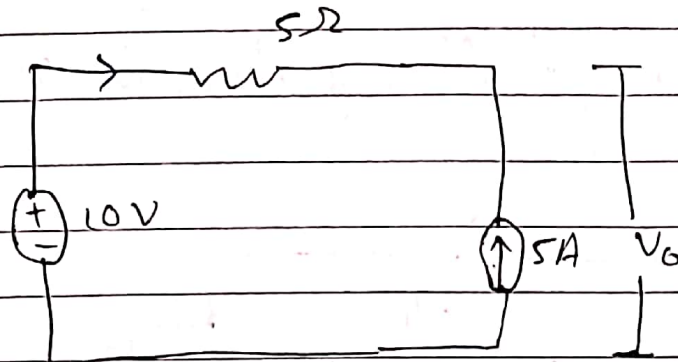
$$\text{The output voltage } (V_o) = 6 - 4 \text{ V}$$

$$= 2 \text{ V}$$

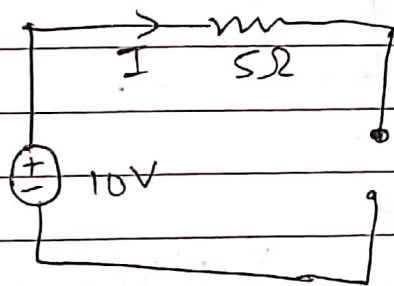
if the

$$V = IR$$

Q With the help of superposition theorem, find

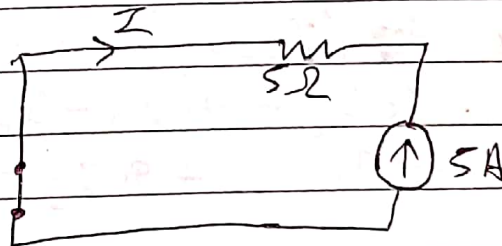


case I) Taking only 10V source



$$\therefore I_1 = 0A \quad V_1 = 10V$$

Case II) Taking only 5A source



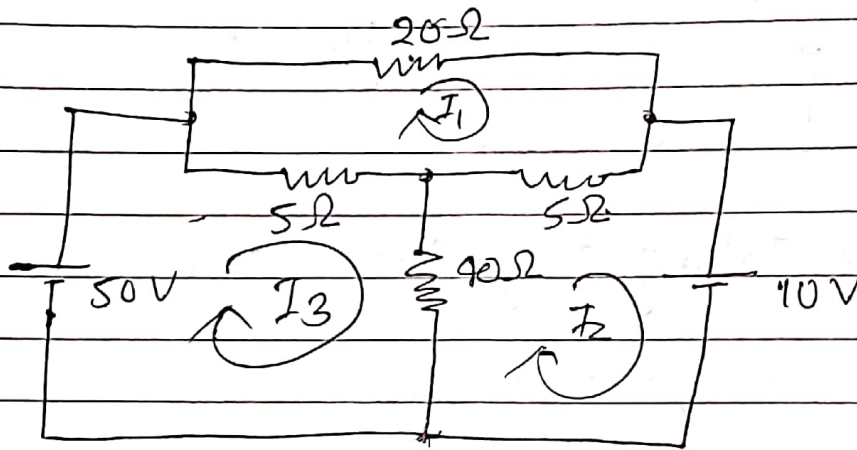
$$I_2 = -5A$$

$$V_2 = 5 \times 5 = 25V$$

$$\therefore \text{Total Current} = (0 - 5) \text{ A} = -5 \text{ A}$$

$$\begin{aligned} \text{Total Voltage } (V_0) &= V_1 + V_2 \\ &= 10 + 25 = 35 \text{ V} \end{aligned}$$

9 Find the current flowing through 40Ω resistance of the circuit shown.



① Mesh analysis;

At I_1 loop

$$-20I_1 - 5(I_1 - I_2) - 5(I_1 - I_3) = 0$$

$$-20I_1 - 5I_1 + 5I_2 - 5I_1 + 5I_3 = 0$$

$$-30I_1 + 5I_2 + 5I_3 = 0 \quad \text{--- (I)}$$

At I_2 loop;

$$-10 - 40(I_2 - I_3) - 5(I_2 - I_1) = 0$$

$$-10 - 40I_2 + 40I_3 - 5I_2 + 5I_1 = 0$$

$$5I_1 - 45I_2 + 40I_3 = 10 \quad \text{--- (II)}$$

At I_3 loop:

$$50 - 5(I_3 - I_1) - 40(I_3 - I_2) = 0$$

$$50 - 5I_3 + 5I_1 - 40I_3 + 40I_2 = 0$$

$$5I_1 + 40I_2 - 45I_3 = -50 \quad \text{--- (III)}$$

Solving eqn ① ② & ③

$$I_1 = 2 \text{ A}$$

$$I_2 = 5.64 \text{ A}$$

$$I_3 = 6.35 \text{ A}$$

~~Q. 7.1~~

Finding current at 40Ω

$$= I_3 - I_2$$

$$= 6.35 - 5.64$$

$$= 0.71$$

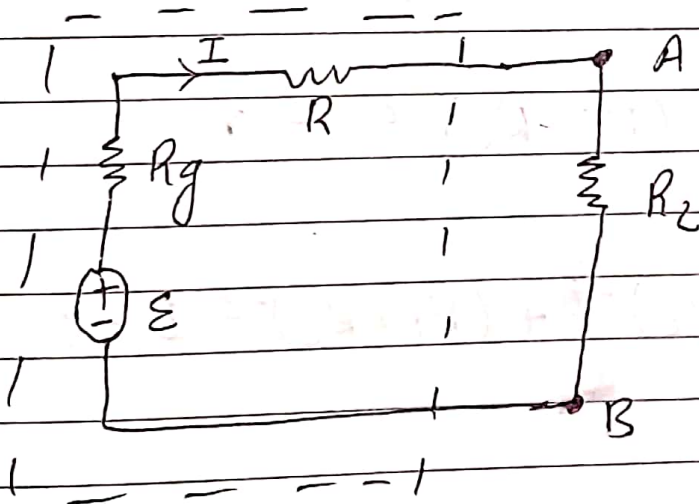
Maximum power Transfer (MPT) Theorem:-

It states that, "A resistive load will abstract maximum power from a network when a load resistance is equal to the resistance of the network as viewed from the output terminals with all energy sources removed having behind their internal resistances."

In figure a load resistance R_L is connected across the terminals A and B of a network which consists of a generator of series resistance R_g and emf $\mathcal{E}$

Let $R_i = R_g + R =$ internal resistance of the network as viewed from the terminals A and B.

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.



In the above circuit

$$\text{current } (I) = \frac{E}{R_i + R_L}$$

$$\begin{aligned} \text{Power dissipated across load } (P_L) &= I^2 R_L \\ &= \left(\frac{E}{R_i + R_L} \right)^2 R_L \\ &= \frac{E^2 R_L}{(R_i + R_L)^2} \end{aligned}$$

For power to be maximum;

$$\text{or, } \frac{dP_L}{dR_L} = 0$$

$$\text{or, } \frac{d}{dR_L} \left[\frac{E^2 R_L}{(R_i + R_L)^2} \right] = 0$$

$$\text{or, } E^2 \frac{d}{dR_L} [R_L (R_i + R_L)^{-2}] = 0$$

$$\text{or, } R_L \cdot -2 (R_i + R_L)^{-3} + (R_i + R_L)^{-2} \cdot 1 = 0$$

$$or, \frac{-2R_L}{(R_i + R_L)^3} + \frac{1}{(R_i + R_L)^2} = 0$$

$$or, \frac{-2R_L + R_i + R_L}{(R_i + R_L)^3} = 0$$

$$or, \frac{R_i - R_L}{(R_i + R_L)^3} = 0$$

$$or, \boxed{R_i = R_L}$$

The maximum power: $P_L(\max) = \frac{E^2 R_L}{(R_i + R_L)^2}$

$$= \frac{E^2 R_L}{(2R_L)^2}$$

$$= \frac{E^2 R_L}{4R_L^2}$$

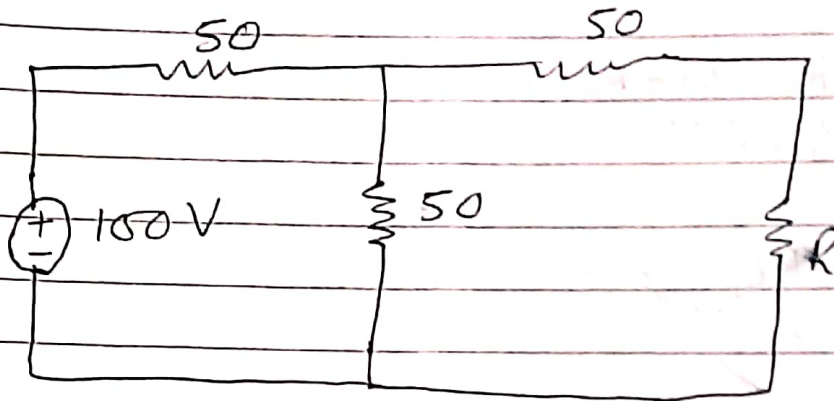
$$\therefore P_L(\max) = \frac{E^2}{4R_L} = \frac{E^2}{4R_i} = \frac{E^2}{4R_m}$$

$$\frac{2500}{300}$$

q.03

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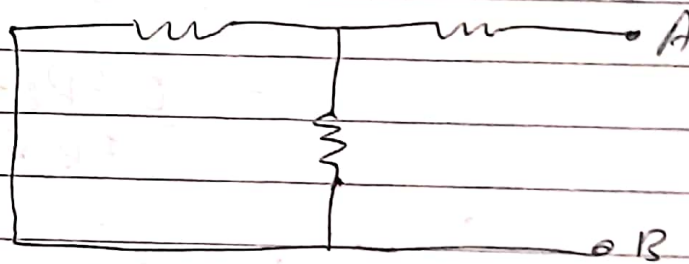
- ③ Find the value of R in the circuit below such that maximum power transfer place through R . Also Calculate the value of power transferred.



Solution

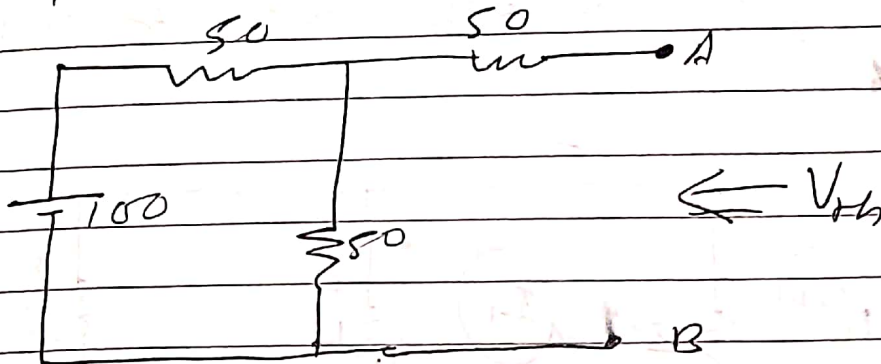
for maximum power transfer;

$R =$ total internal resistance of network
 $=$ Thevenin's resistance as viewed from open terminals removing R and source



$$\begin{aligned} \text{Then } R_{th} &= (50 // 150) + 50 \\ &= 25 + 50 \\ &= 75 \Omega \end{aligned}$$

Now, V_{th}



$$100 - 50I - 50I = 0$$

$$I = 1 \text{ A}$$

V_{th} = Voltage drop across 50Ω

$$= I \times 50$$

$$= 1 \times 50$$

$$= 50 \text{ V}$$

$$\text{maximum power} = \frac{(V_{th})^2}{4R_L}$$

$$= \frac{(50)^2}{4 \times 75}$$

$$= 8.33 \text{ watt}$$

Dependent & Independent Sources:-

Those voltages or current sources which do not depend on any other quantity in the circuit are called independent sources.

For Example: Battery, DC Generator.

Figure shows the ideal and time varying independent voltage & current sources:-

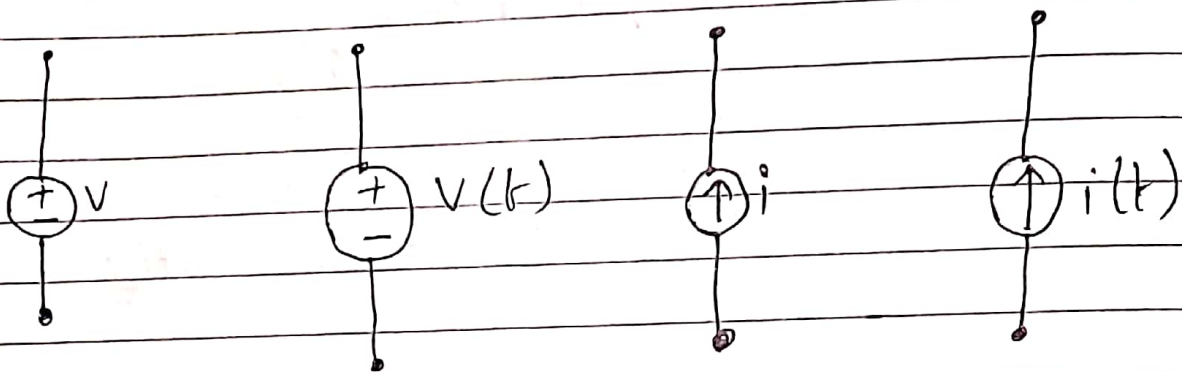


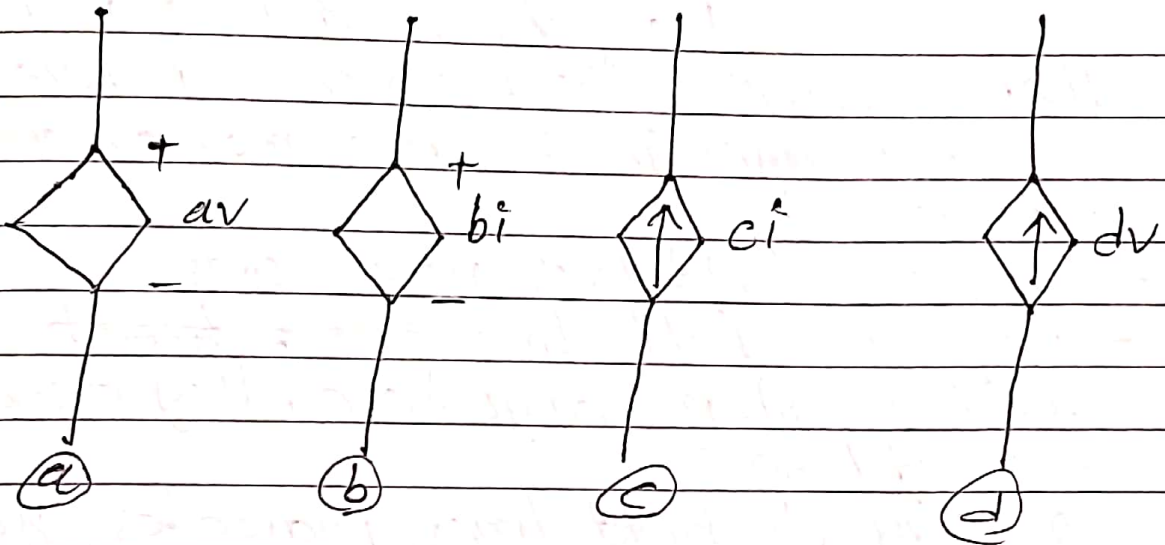
Fig: Independent voltage & current sources

Dependent:-

A dependent voltage or current source is one which depends on some other quantity in the circuit which may be either voltage or current. Such a source is represented by Diamond shaped symbol.

There are four possible dependent sources:-

- (a) Voltage dependent voltage Source
- (b) Current dependent voltage Source
- (c) Current dependent current Source
- (d) Voltage dependent current source



where; a, b, c, d are constants.

a & c has no unit whereas
 b & d are measured in ~~ohms~~.

ohms & mho
respectively.

Unit-1

Magnetism and Electro magnetic Induction

Magnetic field

Magnetic field is a vector field that describes the magnetic influence of electric charges in relative motion and magnetic materials.

- magnetic field lines never cross
- magnetic field lines ~~never don't~~ don't start or stop anywhere, they always make a closed loop.
- density of field lines indicates the strength of the field

④ Magnetic flux:

Magnetic flux is a measurement of the total magnetic field which passes through a given area. Its symbol is ϕ .

The magnetic flux with flat surface area A is given by;

$$\phi = BA \cos \theta, \text{ where;}$$

A = test area

θ = angle between magnetic field vector & surface

B = magnitude of magnetic field vector.

④ Flux density (B)

magnetic flux ~~is~~ density is defined as the amount of magnetic flux in an area taken perpendicular to magnetic flux direction.

If ϕ wb is the total magnetic flux passing normally through area A m² then,

$$B = \frac{\phi}{A} \text{ wb/m}^2 \text{ or Tesla.}$$

⑤ Magnetic field Intensity (H)

Magnetic field intensity is also known as magnetizing force which is measured in Ampere-turns per meter (A-t/m). It is denoted by H .

Magnetic field strength (H) at any point within a magnetic field is numerically equal to the force experienced by a N-pole of one weber placed at that point.

Its unit is also Newtons/weber = N/wb

⑥ Magnetic Permeability (μ)

Magnetic permeability is the ability of a magnetic material to support magnetic field development. The phenomena of magnetism and electro magnetism and development upon a certain

property of the medium called its permeability.

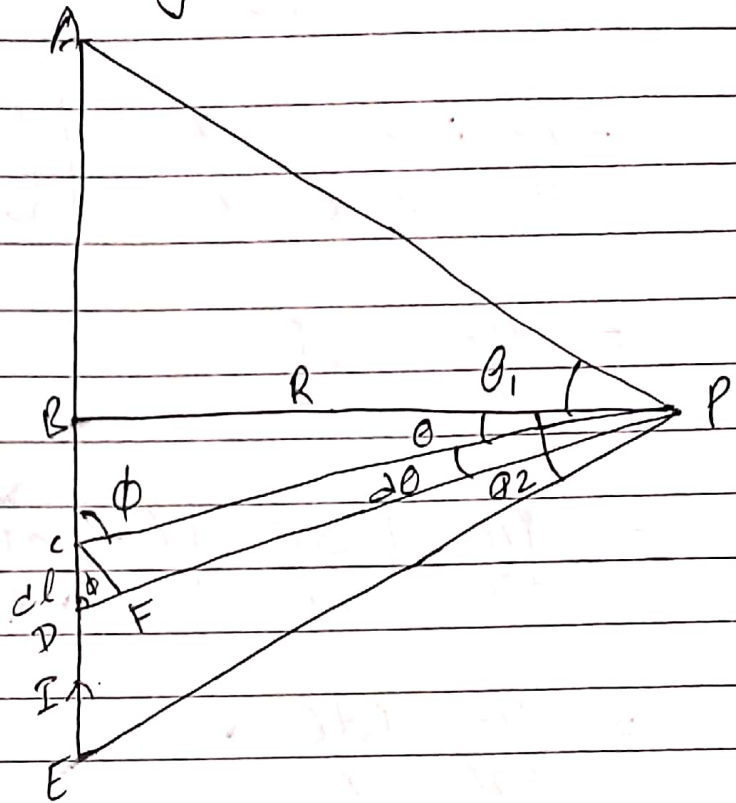
Every medium is supposed to possess two permeabilities.

- ① Absolute permeability (μ_0)
- ② Relative permeability (μ_r)

permeability of free space or ~~vacuum~~ vacuum is constant and denoted by μ_0 and is given by:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m.}$$

④ Magnetic field due to straight current carrying conductor



AE is a straight current carrying conductor. P is a point at normal distance R. We have to find magnetic field at point P due to current i. P is located in such a way that to end A and E it has elevation θ_1 and $-\theta_2$.

θ is the angle made by smallest element dl which makes small magnetic field dB at point P.

Now;

Using Biot Savart law;

$$dB = \frac{\mu_0 i dl \sin \phi}{4\pi r^2}$$

$$= \frac{\mu_0 i \cancel{dl} \sin \phi}{4\pi \sin \phi \cdot r^2} \quad \left[\because \frac{dl}{\cancel{dl}} = \sin \phi \right]$$

$$= \frac{\mu_0 i \cancel{dl}}{4\pi r^2}$$

$$= \frac{\mu_0 i r d\theta}{4\pi r^2} \quad \left(\because \frac{\cancel{dl}}{r} = d\theta \right)$$

$$= \frac{\mu_0 i d\theta}{4\pi r}$$

$$= \frac{\mu_0 i}{4\pi r} d\theta$$

$$= \frac{\mu_0 i \cos \theta}{4\pi R} d\theta \quad \left(\because \cos \theta = \frac{R}{r} \right)$$

Integrating on both side we get;

$$\int dB = \int_{-\theta_2}^{\theta_1} \frac{\mu_0 i \cos \theta}{4\pi R} d\theta$$

$$= \frac{\mu_0 i}{4\pi R} \int_{-\theta_2}^{\theta_1} \cos \theta d\theta$$

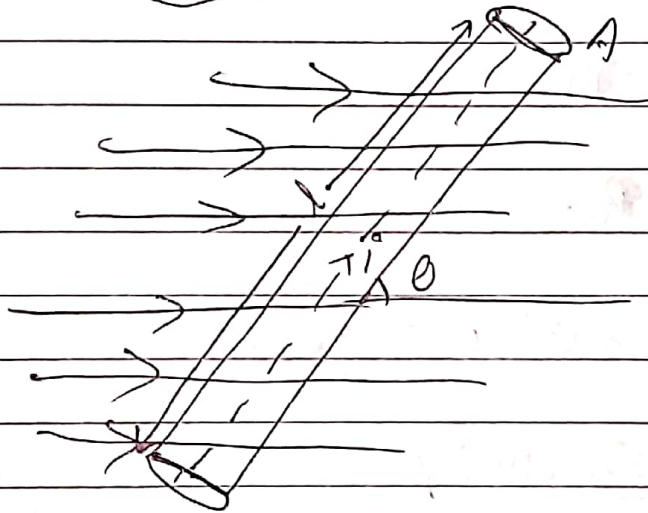
$$= \frac{\mu_0 i}{4\pi R} [\sin\theta]_{-\theta_2}^{\theta_1}$$

$$= \frac{\mu_0 i}{4\pi R} [\sin\theta_1 - \sin(-\theta_2)]$$

$$= \frac{\mu_0 i}{4\pi R} [\sin\theta_1 + \sin\theta_2]$$

$$\therefore B = \frac{\mu_0 i}{4\pi R} (\sin\theta_1 + \sin\theta_2)$$

Force on Current Carrying Conductor in magnetic field



Consider a conductor of length 'l' carrying current 'I' placed at an angle θ with magnetic field B.

In magnetic field when a charge particle is moving, it experience a force

let N be no of free electrons. The force experienced by the conductor is given by :-

$$F = N \times (\text{force experienced by an electron})$$

$$= N e v \sin \theta$$

$$\therefore F = B N e v \sin \theta \quad \text{--- (1)}$$

where we have, $I = n e A v$ where
 n = no of electrons per unit volume
 A = area of cross section

$$I = \left(\frac{N}{V} \right) e A v$$

$$= \frac{N}{A \times l} e A v$$

$$= \frac{N}{l} e v$$

$$\boxed{I l = N e v} \quad \text{--- (2)}$$

From eqn (1) & (2) we get;

$$\boxed{F = B I l \sin \theta}$$

In vector form ; $\vec{F} = I (\vec{l} \times \vec{B})$

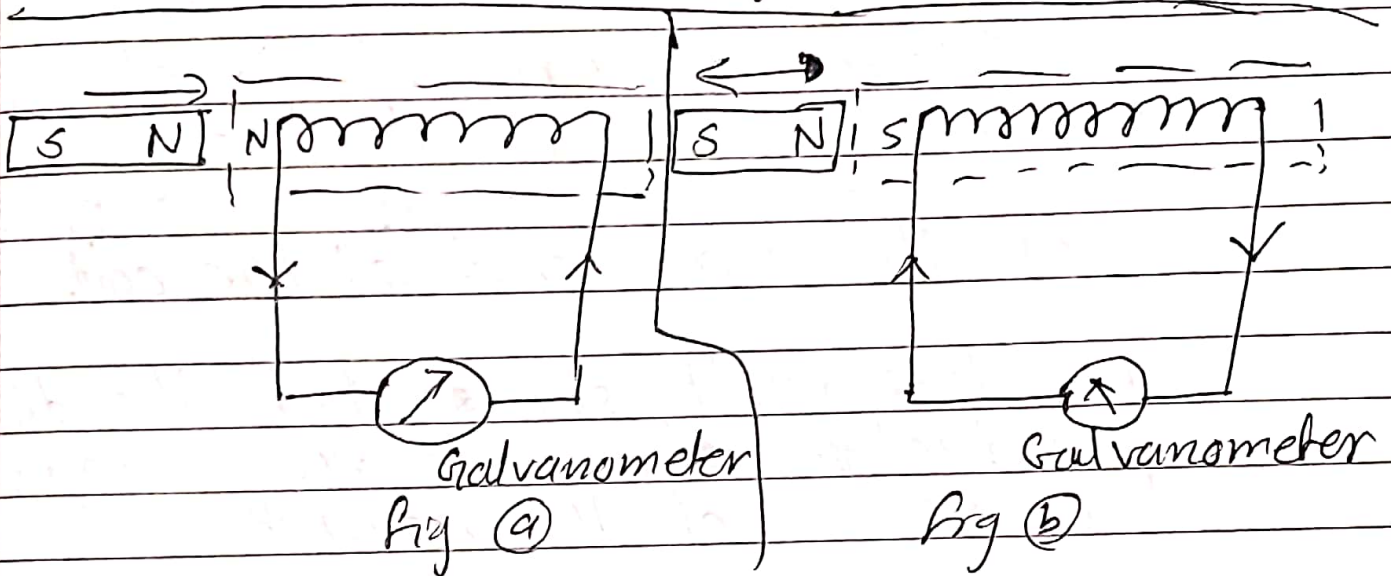
The direction of $\vec{F}$ is given by Fleming's left hand rule.

Special cases:

(i) when $\theta = 90^\circ$; $F_{\max} = BIL$

(ii) when $\theta = 0^\circ$, $F_{\min} = 0$

Lenz law of Electromagnetic Induction



The induced current always flows in such a direction as to oppose the change causing it.

When a magnetic is moved towards and outwards of the coil, the current is induced at that coil. The induced current opposes the motion of the magnet.

Consider a coil connected across the galvanometer. When N pole of the magnetic is brought near to the coil as shown in fig (a), then the

galvanometer is deflected to right. It indicates that the current flows through the coil in anticlockwise direction. When N pole of the magnetic is brought away from the coil as in figure 'b' then the galvanometer is deflected to left. It indicates that the current flows through the coil in clockwise direction and S-pole is developed at the near end of the coil.

These two results show that the direction of induction ~~at~~ circuit always opposes the motion of the magnet.

Faraday's law of Electro magnetic Induction

On the basis of Experiment scientist Faraday proposed two laws:-

- ① An emf induced in the coil so long as there is change in magnetic flux associated with the coil.
- ② The magnetic field of ~~induced~~ induced emf in the coil is directly proportional to the rate of change of magnetic flux associated with the coil.

If $d\phi$ be the change in magnetic flux in time 'dt' then according to law;

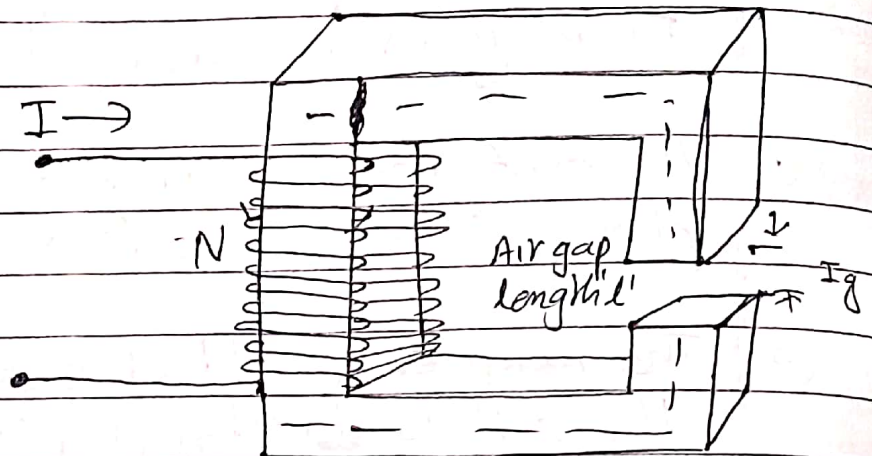
$$\epsilon \propto \frac{d\phi}{dt}$$

$\therefore \epsilon = -k \frac{d\phi}{dt}$ where k is the constant of proportionality and whose value is experimentally.

The negative sign indicates the opposite nature of induced emf.

$$\therefore \boxed{E = \frac{d\phi}{dt}} \text{ in magnitude.}$$

Magnetic circuit concept



The flux producing ability of the coil on any ~~perpendicular~~ magnetic ~~flux~~ field is proportional to the no of turns N and current I .

The product NI is called the magnetomotive force (mmf) and determines the amount of flux developed in the magnetic circuit.

$$\text{MMF} = NI \text{ ampere-turns.}$$

Total ~~amount~~ flux developed in the circuit is given as, $\phi = \frac{\text{MMF}}{S}$ webers

where; S is the reluctance of the magnetic circuit.

Magnetic Hysteresis:

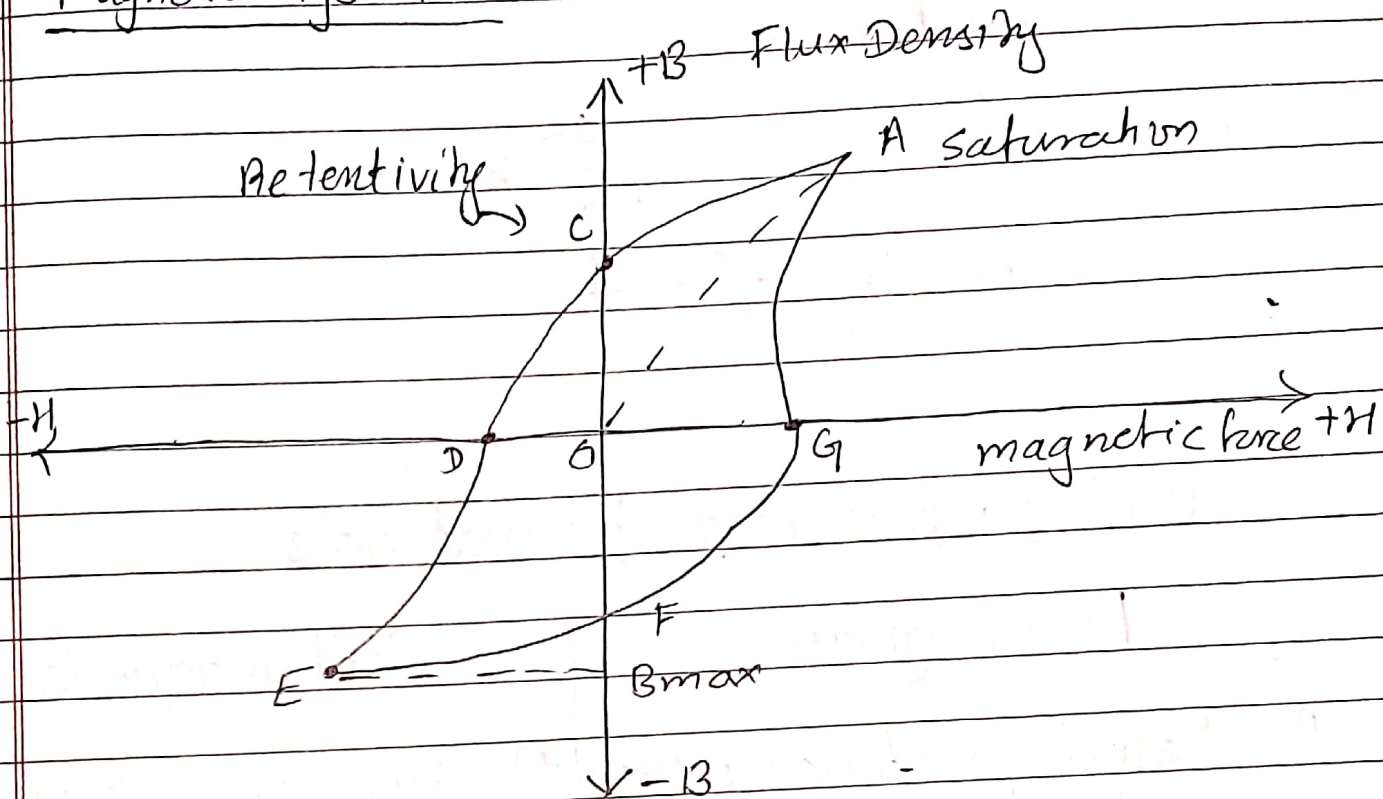


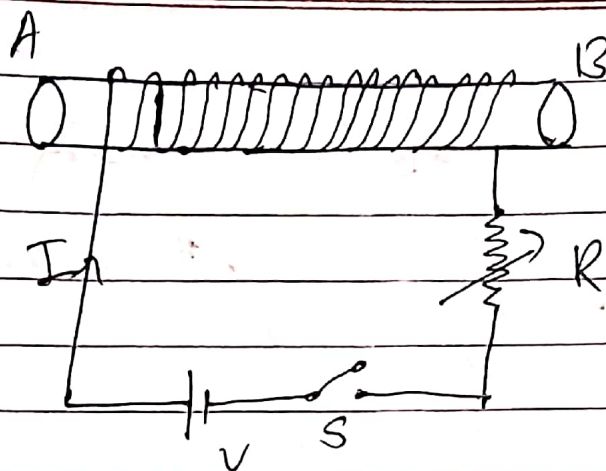
Fig: Typical magnetic Hysteresis for ferromagnetic material

It is define as the ~~gr~~ quality of magnetic substance due to which is dissipated in it on the ~~re~~ reversal of its magnetism.

→ The value ~~is~~ B is increased or decreased by increasing or decreasing through the coil.

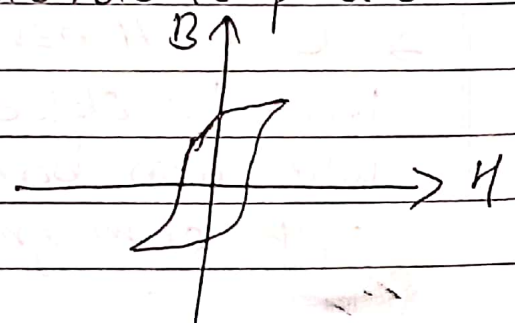
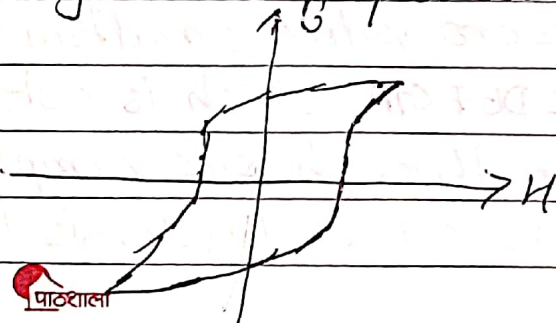
→ If we plot a relation between B and H a curve OA is obtained.

→ B and H never attain zero value simultaneously. When the closed loop $ACDEFGA$ which is obtained when iron bar is taken through one complete cycle of magnetization is called hysteresis loop.



Hard and Soft magnetic materials

Hard magnetic	Soft magnetic
1) Magnetic material which retain their magnetism and are difficult to de-magnetized are called hard magnetic materials.	1) Soft magnetic materials are easy to magnetized and demagnetized.
2) They are used to make permanent magnet.	2) They are used to make temporary magnet.
3) They have large hysteresis loop area.	3) They have small hysteresis loop area.



4) Susceptibility & permeability are low.

5) Coercivity & retentivity are high.

6) Magnetic energy stored is high.

7) Eddy current loss is high.

4) Susceptibility & permeability are high.

5) Coercivity & retentivity are low.

6) Magnetic energy stored is low.

7) Eddy current is low.

Unit-7

Cell & batteries

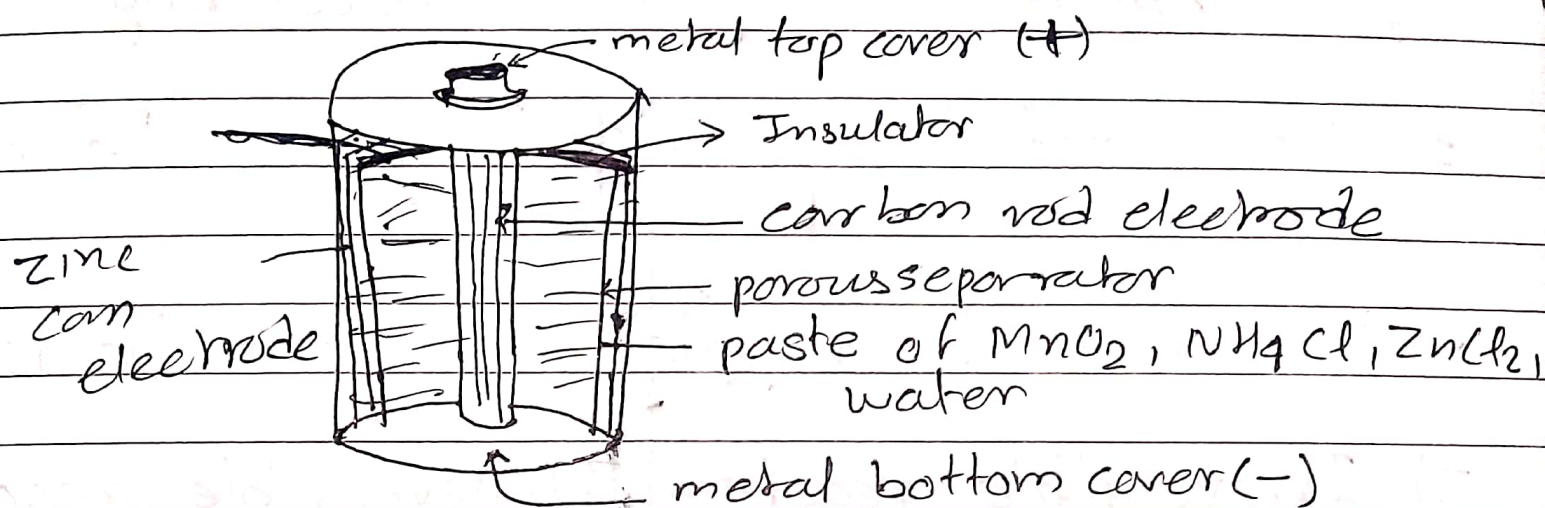
Primary cells:-

Primary cells are those cells that cannot be recharge and need to be discarded after the expiration of their life time. Primary cells usually have high energy density, capacity, are slowly to discharge, easy to use and not excessively expansion. Alkaline are probably most commonly used primary batteries. They usually have zinc-anode, carbon-cathode & electrolyte.

Secondary cells:-

Secondary cells are those that can be recharged after usage or discharge and it is possible to use them several times.

They have quick discharge rate and need to recharged again and again. These cells are initially costly than primary cells. All mobile phones are used this types of cells. The most commonly use secondary cell are lithium-ion (Li-ion), Nickel-cadmium (Ni-cd) cells.



Primary	Secondary
1) They cannot be recharged.	1) They can be recharged.
2) They cannot be reused.	2) They can be reused.
3) Irreversible reaction occurs.	3) Reversible reaction occurs.
4) Smaller and lighter design.	4) More complex and heavier design.
5) They are generally cheap.	5) They are initially costly.
6) Used in portable devices such as radio, watch etc.	6) They are used in automobile, inverter and mobile phone.
7) Zinc-carbon cells.	7) Nickel-Cadmium (Ni-Cd) cells.

Internal resistance of battery

Internal resistance is the resistance present within the battery that resists the current flow when connected to a circuit thus it causes a voltage drop when current flow through it. It is the resistance provided by the electrolyte & electrodes which is present in the cell.

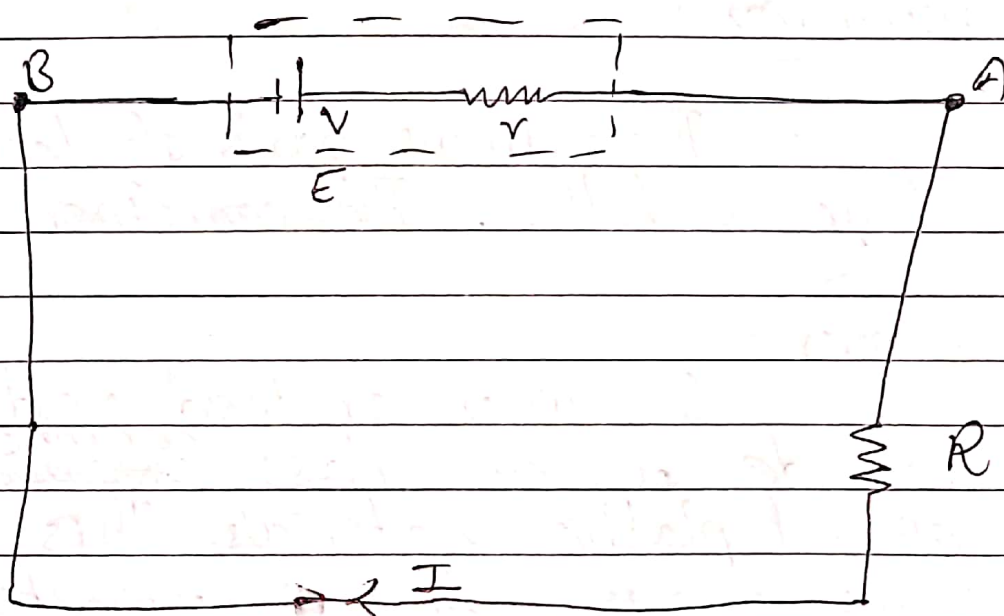


Fig: A battery of emf E & resistance r connected to load R .

Here;

$$E = V + Ir$$

where; r is the internal resistance of battery.

Lead acid Battery (LAB)

⇒ The battery which uses sponge lead and lead peroxide for the conversion of chemical energy into electrical energy is called lead acid battery.

The lead acid battery is most commonly used in powerstations because it has higher cell voltage and lower cost.

Construction:

The various parts of lead acid battery are as follows. The ~~container~~

① Container:

Container of lead acid battery is made up of glass, ~~ceramic~~ ^{ceramic} materials & special plastic materials. This contains chemical energy which is converted into electrical energy.

② Plates:

The plates are two types:

① -ve

② +ve

The positive ones consist lead peroxide and a negative ones consist sponge "

A battery has two terminals taken from each plate.

③ Separators:

They are thin sheets of non-conducting materials which are placed between positive & negative plates.

④ Electrolytes:

Dilute sulphuric acid is used as electrolytes. It contains nearly 31% of H_2SO_4 .

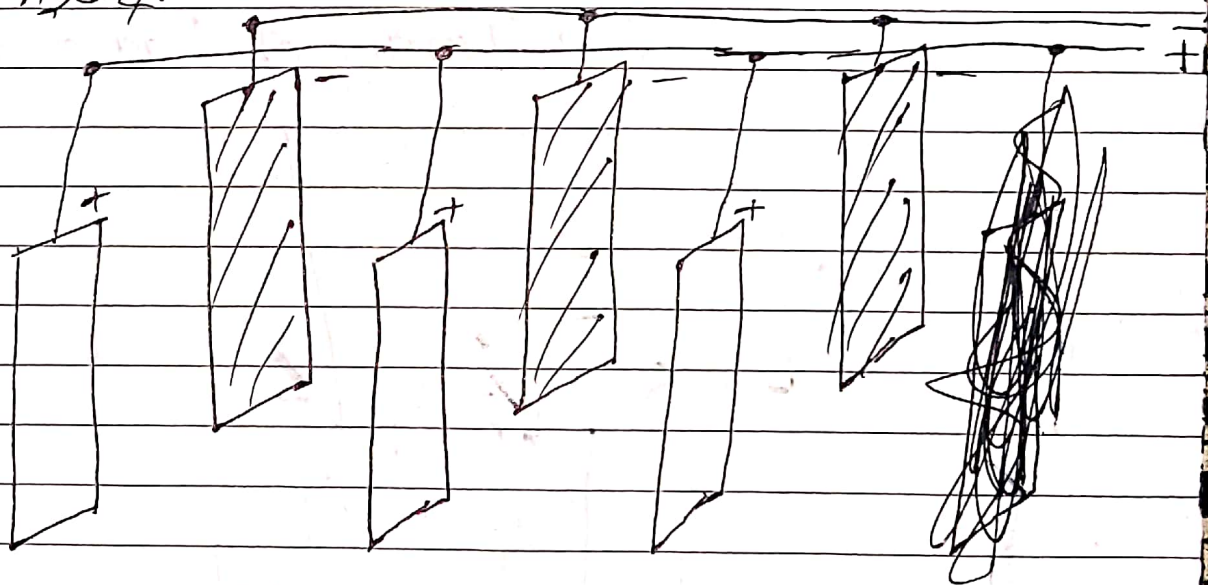


Fig: Arrangement of cells in lead acid battery

Charging & Discharging

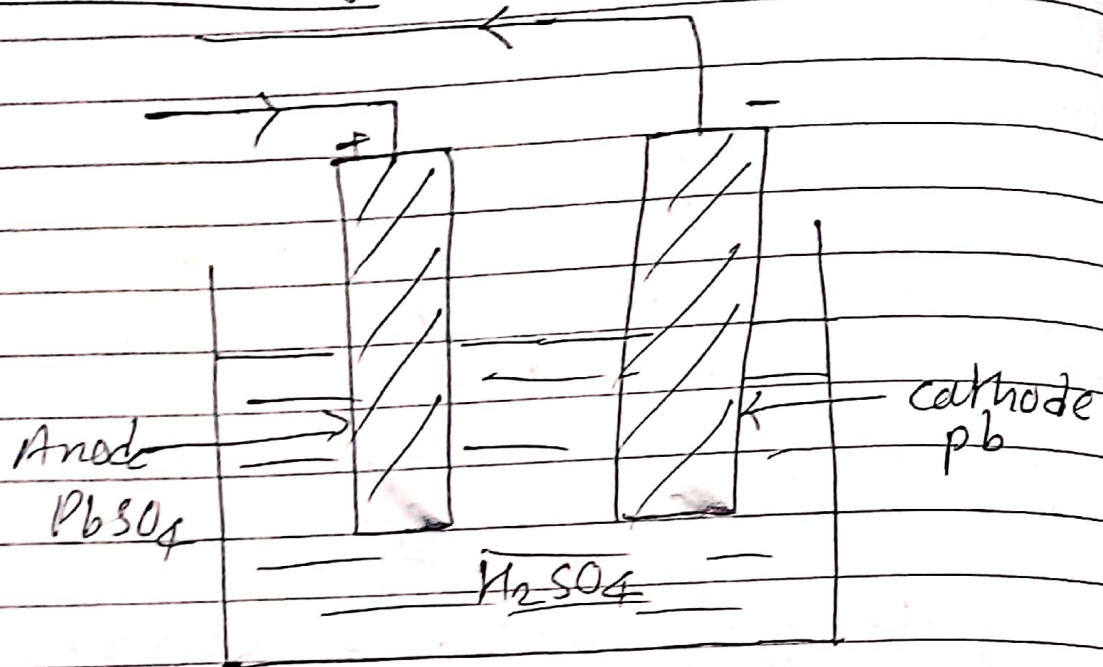


Fig: Charging of lead acid battery.

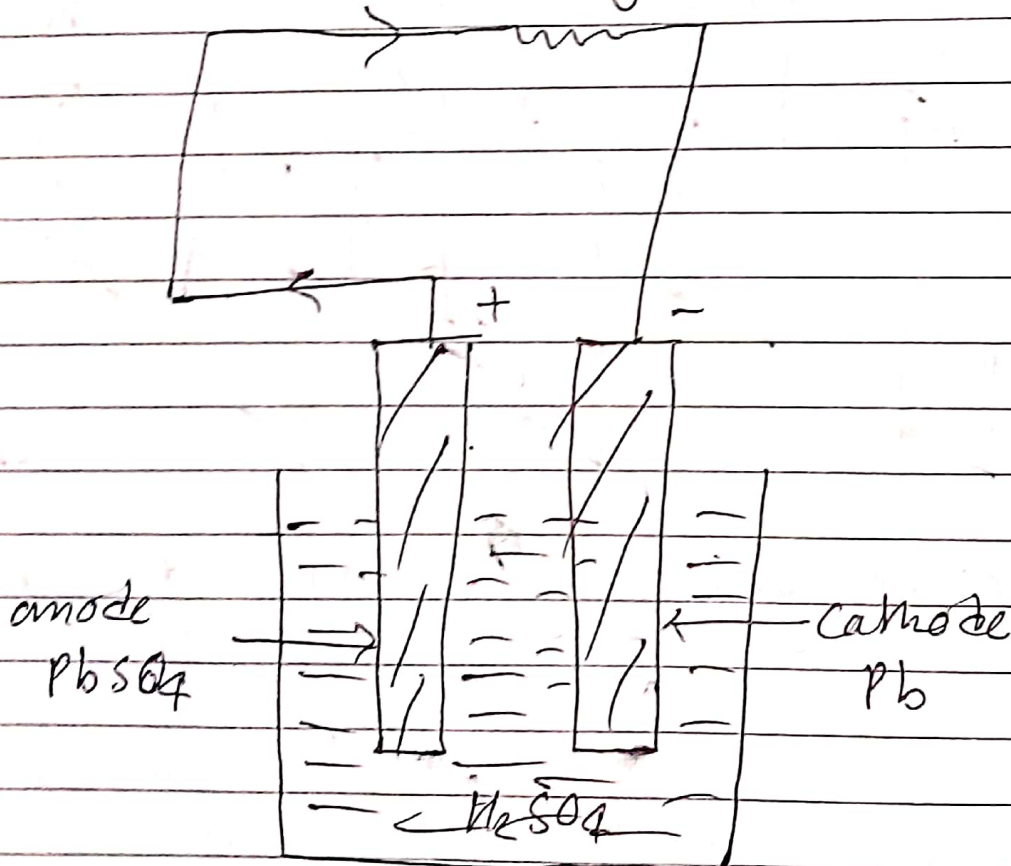
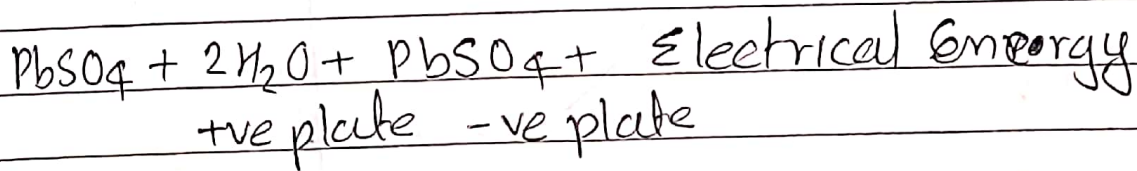
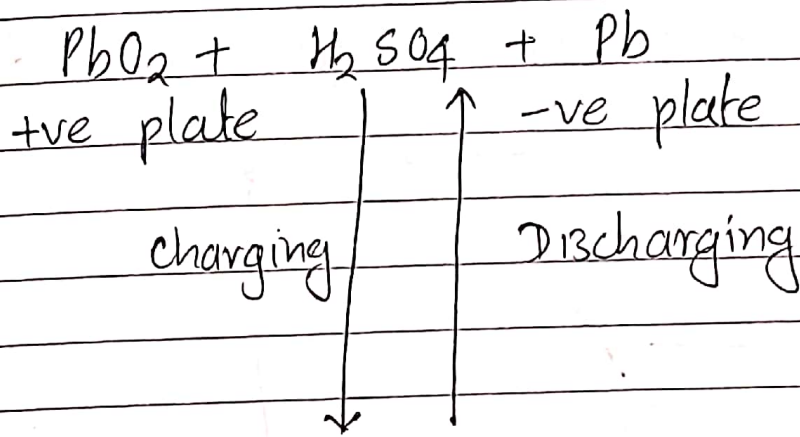


Fig: Discharging of lead acid battery.

The charging and discharging are represented by a single reversible equation given below.



Lead Acid Battery Charging methods

The lead acid battery uses two types of charging methods. They are:-

- ① Constant Voltage charging
- ② Constant Current charging

① Constant Voltage charging:-

It is most common method of charging lead acid battery. It reduces the charging time & increasing the capacity but reduces the efficiency.

In this method, the charging voltage is kept constant throughout the charging process.

The charging current is high in the beginning when the battery is in the discharge condition. The current is gradually dropping off ^{as} ~~at~~ the battery picks up charge.

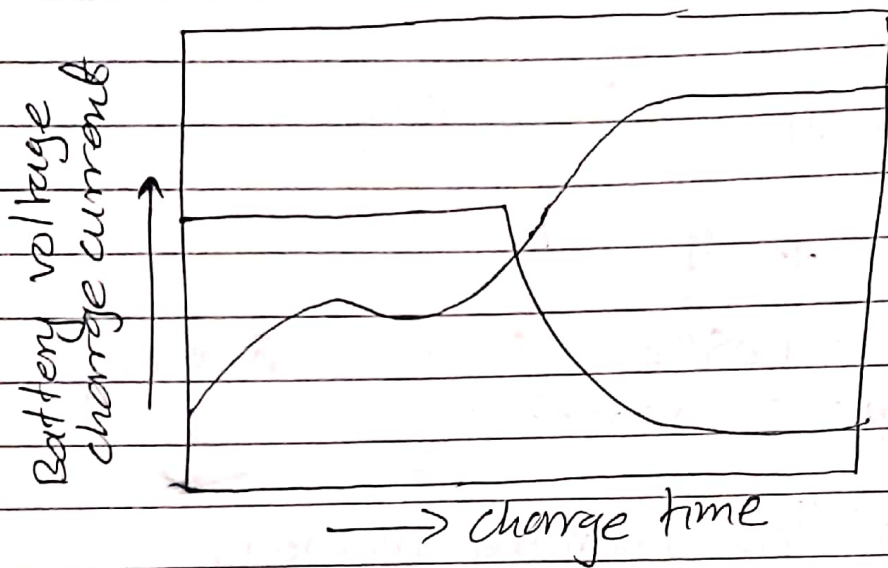
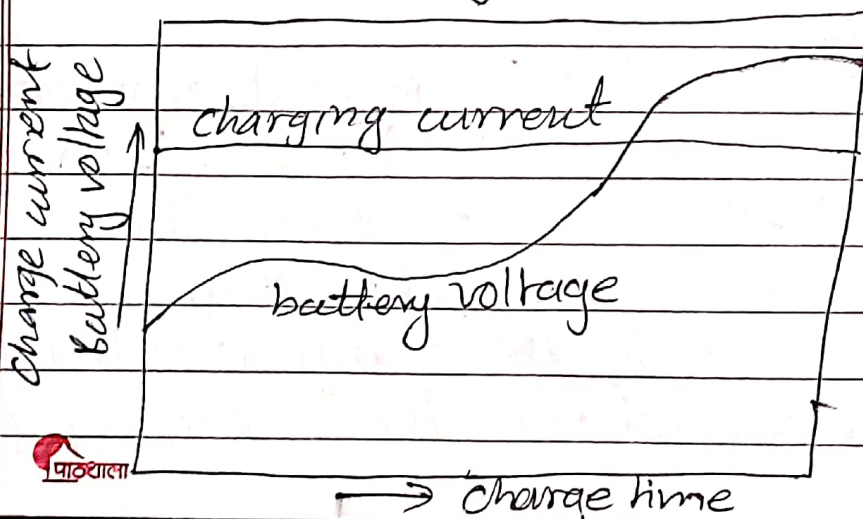


Fig: constant voltage charging

① Constant current charging:

In this method, the charging current is kept constant throughout the charging process. In this method, charging batteries are connected in series so as to form a group and each charges from DC supply.



7.3, Dry cell, Mercury cell, Ni-Cd Cell, Li-ion cell

Dry cell:

A dry cell is an electrochemical cell. It has the electrolyte immobilized as a paste with only enough moisture in it to allow current to flow. A dry cell can operate in any orientation.

The common dry cell battery is zinc carbon battery. This is made of an outer zinc container which acts as anode and a carbon rod acts as cathode. The electrolyte is paste of ammonium chloride (NH_4Cl).

Mercury cell, Ni-Cd cell and Li-ion cell are other common types of dry cell.

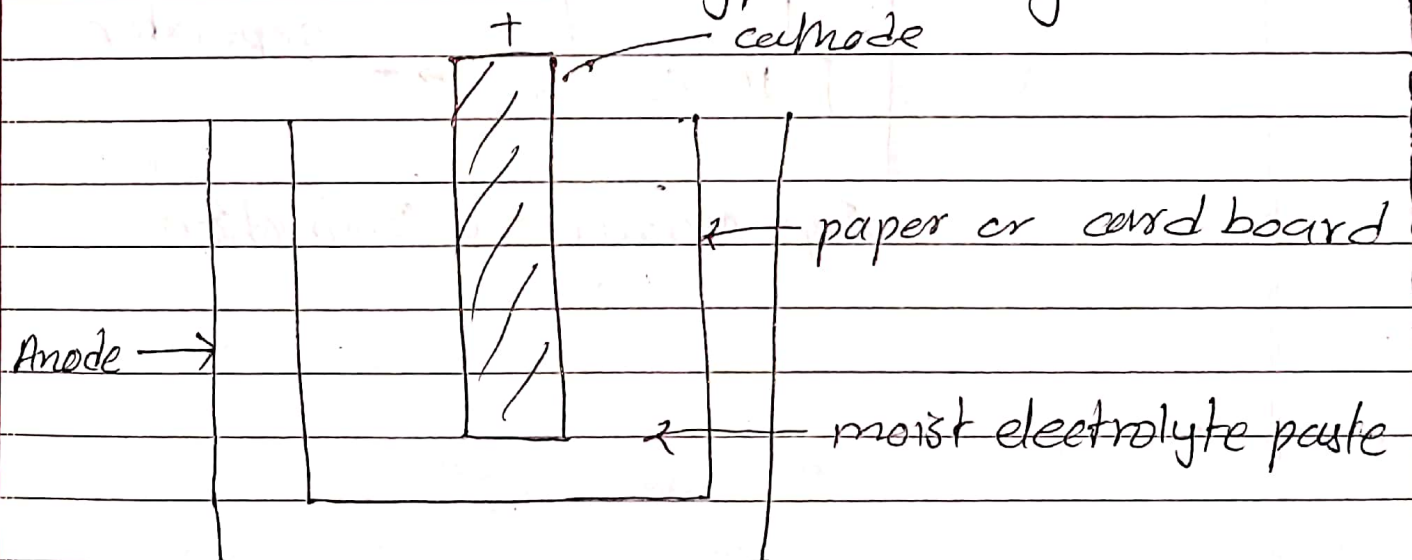


Fig: Dry cell

Mercury Cell

A mercury cell is non rechargeable electrochemical battery. They are generally used in watches, hearing aids, calculator and computers. They have advantage of long life upto 10 years and steady voltage output.

Mercury battery uses either pure mercuric oxide or mixture of mercuric oxide with manganese dioxide as the cathode and potassium hydroxide as an electrolyte.

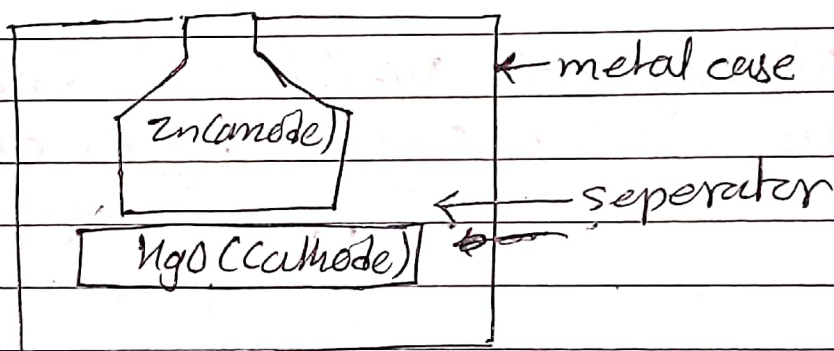
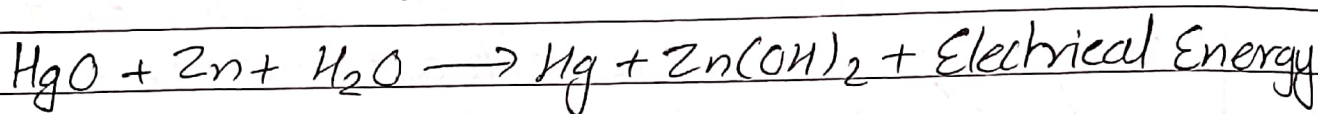


Fig: Mercury cell Construction

Ni-Cd or Ni-Cad Cell

Nickel-Cadmium battery is a type of rechargeable battery using Nickel oxidation hydroxide and metallic Cadmium as electrodes. Ni-Cd batteries have a very long ~~to~~ operating life.

These types of cell are commonly used in portable computers and mobile. Ni-Cd cells are relatively easy to manufacture but it is very toxic. Additionally, Nickel and Cadmium are very expensive.

Ni-Cd cells can supply extremely high currents and can be recharged rapidly.

Li-ion Battery:-

A Lithium ion battery or Li-ion battery is a type of rechargeable electrochemical cells. They are commonly used for mobiles, laptops and cameras. They are easy to handle and portable.

Li-ion batteries are the powerhouse for digital electronics evolution. The high energy density and large number of discharge cycle are most important factors of this

battery.

Li-ion battery are more expensive than Ni-Cd battery but operate over a wide range of temperature with high energy density. They are less toxic than Ni-Cd battery. Metallic lithium is used as electrodes.

* Serial and parallel Connection of cells

* Serial Connection of cells:-

Cells are joined end to end so that the same current flows through each cell. such type of connection is called serial connection of cell. -

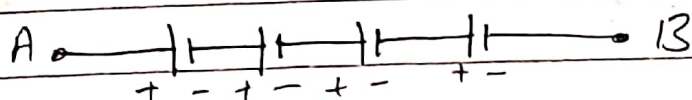
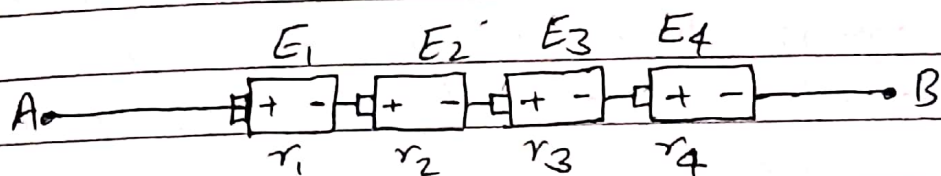
In this case, ~~the~~ if the cells are connected in series the emf of the battery is connected to the sum of emf of the individual cells.

If $\mathcal{E}$ is the overall emf of the battery ~~com~~ combined with n no of cells and $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \dots, \mathcal{E}_n$ are the emf's of cell then;

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \dots + \mathcal{E}_n$$

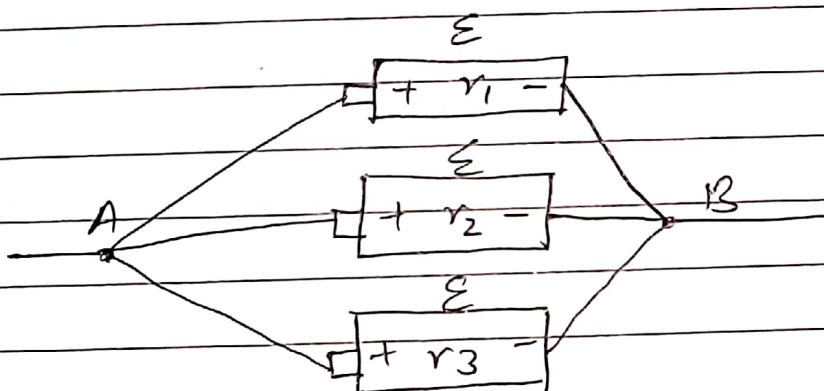
$$r = r_1 + r_2 + r_3 + \dots + r_n$$

where; r = internal resistance.



* Parallel Connection of Cells :-

In a parallel connection, all positive terminals are connected together and negative terminals are connected together.



$$E = E_1 = E_2 = E_3 = E_n$$

Similarly, the internal resistance is calculated as;

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots + \frac{1}{r_n}$$

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② Hybrid Connection

If both series connection and parallel connection is implemented in a circuit ^{from} such that it is called a hybrid connection.

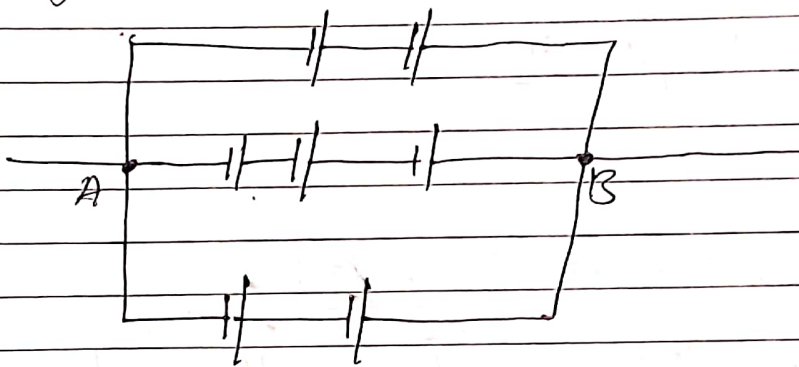


Fig: Hybrid Connection of cells.

Magnetic Circuit

Electric Circuit

1) Flux = $\frac{\text{m.m.f}}{\text{reluctance}}$

1) Current = $\frac{\text{emf}}{\text{resistance}}$

2) M.M.F (ampere-turns)

2) E.M.F (volts)

3) Flux ϕ (webers)

3) Current I (amperes)

4) Flux density B (Wb/m^2)

4) Current density (A/m^2)

5) Reluctance $S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$

5) resistance $R = \rho \frac{l}{A}$

6) Permeance = $(1/\text{reluctance})$

6) Conductance (= $1/\text{resistance}$)

7) Reluctivity

7) Resistivity

8) Permeability (= $1/\text{reluctivity}$)

8) Conductivity (= $1/\text{resistivity}$)

9) Total m.m.f = $\phi S_1 + \phi S_2 + \dots$

9) Total emf = $IR_1 + IR_2 + IR_3 + \dots$

